

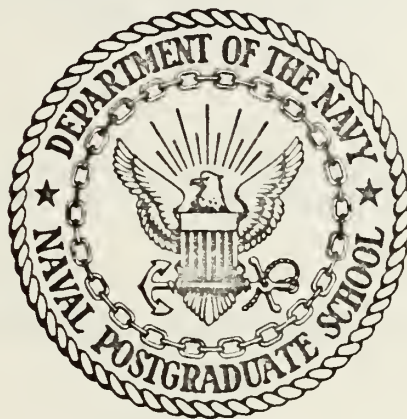
MANEUVERING CONTROL OF REPLENISHMENT
AT SEA

Theodoros Sarzetakis

LIBRARY
NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIF. 93940

NAVAL POSTGRADUATE SCHOOL

Monterey, California



THESIS

MANEUVERING CONTROL OF REPLENISHMENT AT SEA

by

Theodoros Sarzetakis

Thesis Advisor:

G. J. Thaler

September 1972

Approved for public release; distribution unlimited.

T149351

Maneuvering Control of Replenishment at Sea

by

Theodoros Sarzetakis
Lieutenant, Hellenic Navy
B.S., Naval Postgraduate School, 1972

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
September 1972

Thesis
5 of
c. 7

ABSTRACT

An investigation of the maneuvering control of ships involved in the replenishment at sea operation under calm water conditions is carried out.

The linearized differential equations of motion of a vessel in the horizontal plane are established and implemented for the formation of computer programs, useful for the study of the behavior and stability of the ship with and without the influence of control surfaces (rudders).

Three methods of controlling automatically the maneuvering of two ships, in replenishment at sea, under the influence of interactive forces and moments, based on the classical feedback control theory are presented, compared and conclusions are finally drawn about the efficiency of these methods.

TABLE OF CONTENTS

I.	INTRODUCTION	6
II.	THE INTERACTION EFFECTS	8
	A. NEWTON'S EXPERIMENT	8
III.	EQUATIONS OF MOTION	14
	A. THE GENERAL CASE	14
	B. THE HORIZONTAL PLANE MOTION	16
	1. Linearization of the Horizontal Plane Equations	17
	2. Nondimensionalization of the Horizontal Plane Equations	19
	3. Manipulation of the Equations for Computer Simulation	21
	4. Stability Investigation	25
	5. The Transfer Functions	31
IV.	THE CONTROL LOOPS	35
	A. PATHKEEPING LOOP	35
	1. The Block Diagram	35
	2. Determination of K and K_t for a Desired Performance	35
	3. Computer Simulation of the Controlled Plant	38
	B. THE DISTANCE KEEPING LOOP	43
	1. A Special Case	43
	2. Computer Program for the Controlled Plant	46
	3. Determination of K and K_t for a Desired Performance	48
	4. Computer Simulation	50

V.	ACTUAL REPLENISHMENT AT SEA PROBLEM	-----	57
A.	INCLUSION OF INTERACTION FORCES AND MOMENTS	--	57
1.	Characteristic Values	-----	57
2.	Modification of the Equations of Motion	--	59
3.	The Behavior of the Ships Under the Influence of Interacting Forces and Moments Without Controls	-----	63
B.	THE PROPOSED METHODS OF CONTROL	-----	63
1.	Method I	-----	63
2.	Method II	-----	66
3.	Method III	-----	69
VI.	CONCLUSIONS	-----	87
	LIST OF REFERENCES	-----	108
	INITIAL DISTRIBUTION LIST	-----	109
	FORM DD 1473	-----	110

ACKNOWLEDGEMENT

The author wishes to express his deep respect and appreciation to Dr. George Thaler, under whose warm and wise guidance this thesis was completed. He is also deeply indebted to A. T., the great man and real friend, who made it all possible.

I. INTRODUCTION

Replenishment-at-sea operations are conducted for the purpose of safe transferring of the maximum amount of cargo in a minimum of time between ships of the fleet, in order to enable them to operate at sea for prolonged periods. As the cargo must be guided and controlled during the transfer operation, a suitable physical connection (rigs) must be established and maintained between the two ships as they travel along with identical speeds. This connection requires that the ships operate at close quarters, a fact that makes maneuvering during replenishment a critical and dangerous operation.

The maneuver involves six factors: course, speed, distance between ships, the approach, station keeping and departure.

In this thesis the very important phase of maintaining station will mainly be considered. Maintaining station alongside the delivery ship requires precise maneuvering on the part of the receiving ship. Steaming too close restricts maneuverability and steaming too far apart puts an undue strain on the rigs. Steaming too close increases also the turbulence between the ships.

Maneuvering at close proximity does not really by itself present such a great navigational problem, but steaming alongside results in certain hydrodynamic phenomena that create unpredicted interactive forces and moments between the ships that generate the ever existing danger of collision.

Replenishment of warships (mainly with fuel from tankers), is a procedure which is nowadays regarded as commonplace and accompanied by little risk once the vessels have taken up station close aboard. Yet the fact that collisions in open deep water occurred and they still unfortunately do, in spite of modern navigational aids, gives rise to the question as to what part interaction affects may have played in the accidents.

It has been already stated that maintaining station alongside requires precise maneuvering on the part of the receiving ship. This, according to the tactical requirements of the operation is interpreted as follows:

The replenishing ship is responsible for course keeping only.

The receiving ship is responsible for both course and station (distance between ships) keeping.

On this very basis, this thesis attempts the investigation of the automatic maneuvering control of two individual ships in a replenishment at sea operation, under calm water conditions. It first presents the interaction affects problem, and then it establishes the equation of motion for one ship and the development of its mathematical model. Next, it investigates the stability and behavior of the individual ship in motion without controls and proceeds with the presentation and investigation of two major control loops — the course keeping loop and the distance keeping one. Finally, those loops are implemented in certain ways in a somewhat realistic replenishment at sea problem for three different proposed techniques of the operation for which computer simulation results are presented.

II. THE INTERACTION EFFECTS

A. NEWTON'S EXPERIMENT

When underway there are areas of increased water pressure at the bow and stern of a ship, and decreased pressure (suction) amid ships as the result of the differences in velocity of the flow of the water around the hull. When the ships are alongside each other underway, this venturi affect is increased and becomes further complicated because of the intermingling of the pressure areas of the two ships. These effects vary with the distance between ships, size, and configuration of ships, speed, depth of water and sea conditions. Changes in relative position between ships will impose rapid changes in the pressure effects on their hulls. The danger is increased if speed is reduced and radical speed changes will further aggravate the situation. In shallow waters pressure effects are more pronounced and extra care is required in maneuvering (in depths of less than twenty fathoms). It is therefore evident that to maintain station while alongside a certain amount of rudder, is usually necessary and this amount of rudder will vary with the size and load of both ships, sea and wind conditions, speed and ship separation. As a result of such increased rudder, speed is reduced which complicates the problem of maintaining stations, because it increases the handling difficulties of the ships, and it is also dangerous if a rudder casualty should occur.

The classic and original work on the reaction of vessels underway and in close proximity to one another, was the investigation carried out by the late Rear Admiral D. W. Taylor, USN [1], the results of which he presented to the Society of Naval Architects and Marine Engineers in June 1909.

The problem has been studied theoretically by Silverstein [2] and experimentally by Newton [3], with both approaches showing agreement as far as the major trends are concerned. Newton's experiments proved one important thing: It is the process of taking up or breaking away from the abeam or "fueling" position that presents the navigational risks. The experiments were conducted first with 1/50 scale models of real ships that were towed without propellers on parallel courses at different positions relative to each other longitudinally, over a range of corresponding speeds from 10 to 20 knots and at separations 50 and 100 feet, beam-to-beam.

Then full-scale trials in open sea were performed with self-propelled ships. The results of both methods were quite comparable.

When two ships pass close by on parallel courses, the pressure fields mix and the effect is to produce an unbalanced force and moment on each ship, which must be counteracted by the rudder for each to maintain course or avoid collision. Figure 1 shows the measured Y-force and N-moment acting on each model in different positions. The longitudinal separation scale shown as the abscissa is measured between the middle length of the two ships. The rudder angle data shown in Fig. 1 were the ones needed to maintain equilibrium at each of the

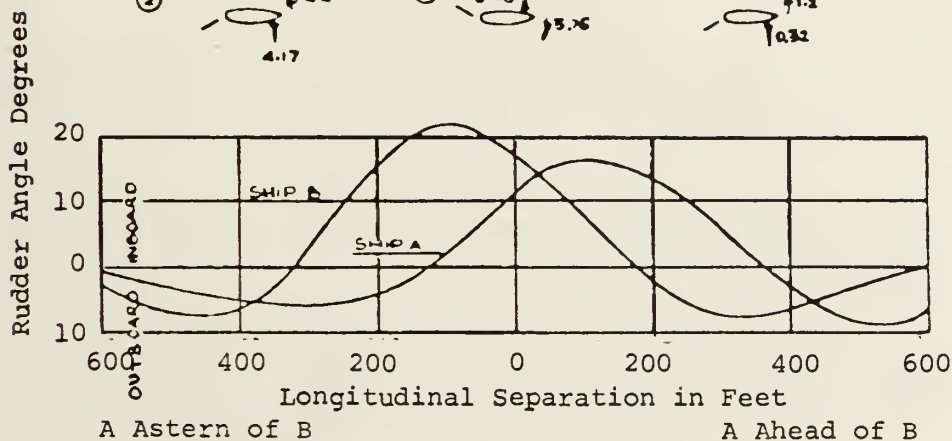
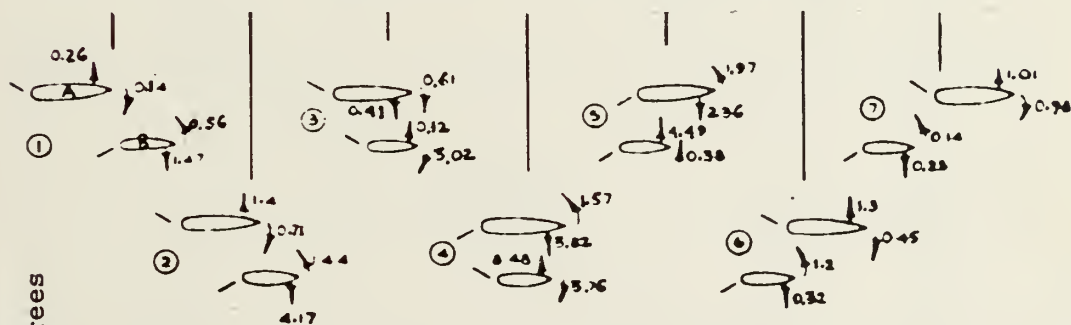
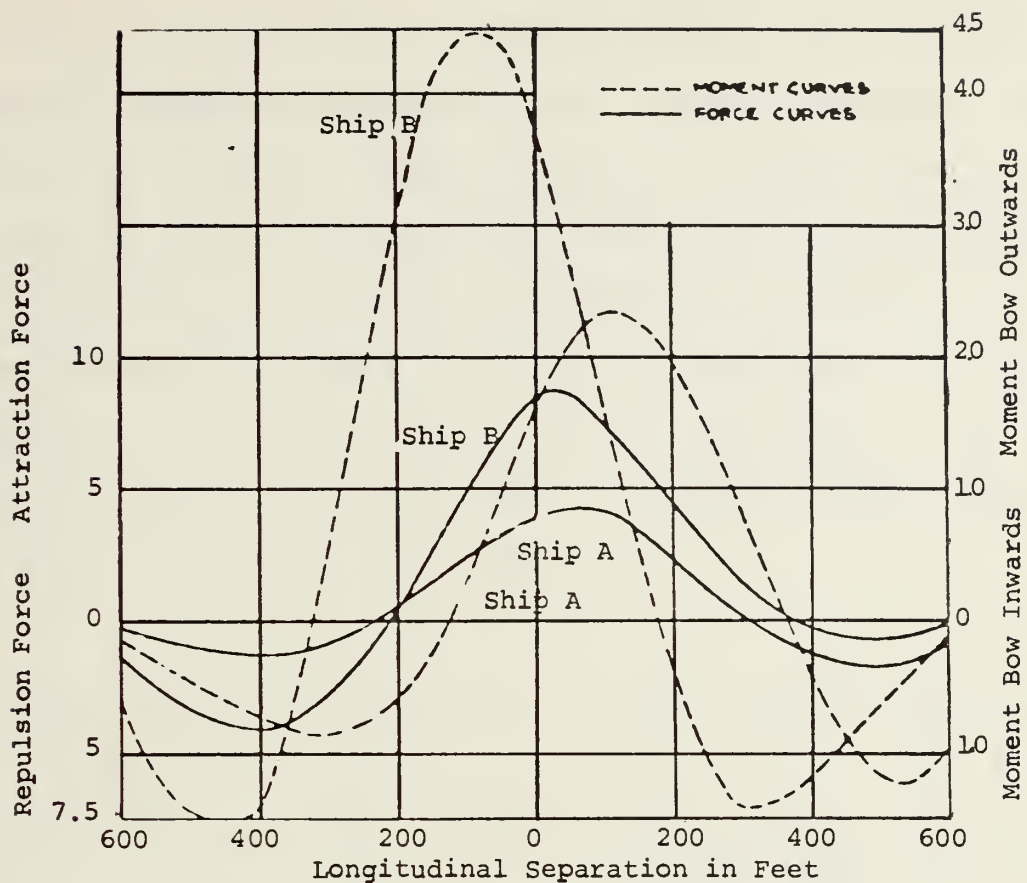


Figure 1. Measured Interaction Forces and Moments and Correcting Rudder Angles

Fifty-foot separation beam-to-beam.

relative positions shown. The magnitudes of the maximum forces of attraction shown in Fig. 1 are of interest. At a speed of 10 knots the maximum attraction force is 26 tons for ship A and 35 tons for ship B for the 50-foot beam-to-beam separation, and these occur when the two ships are very close to the fully abeam position 4. These forces should be quadrupled at a speed of 20 knots and according to Fig. 2 would be decreased by about 40% if the beam-to-beam separation were increased to 100 feet.

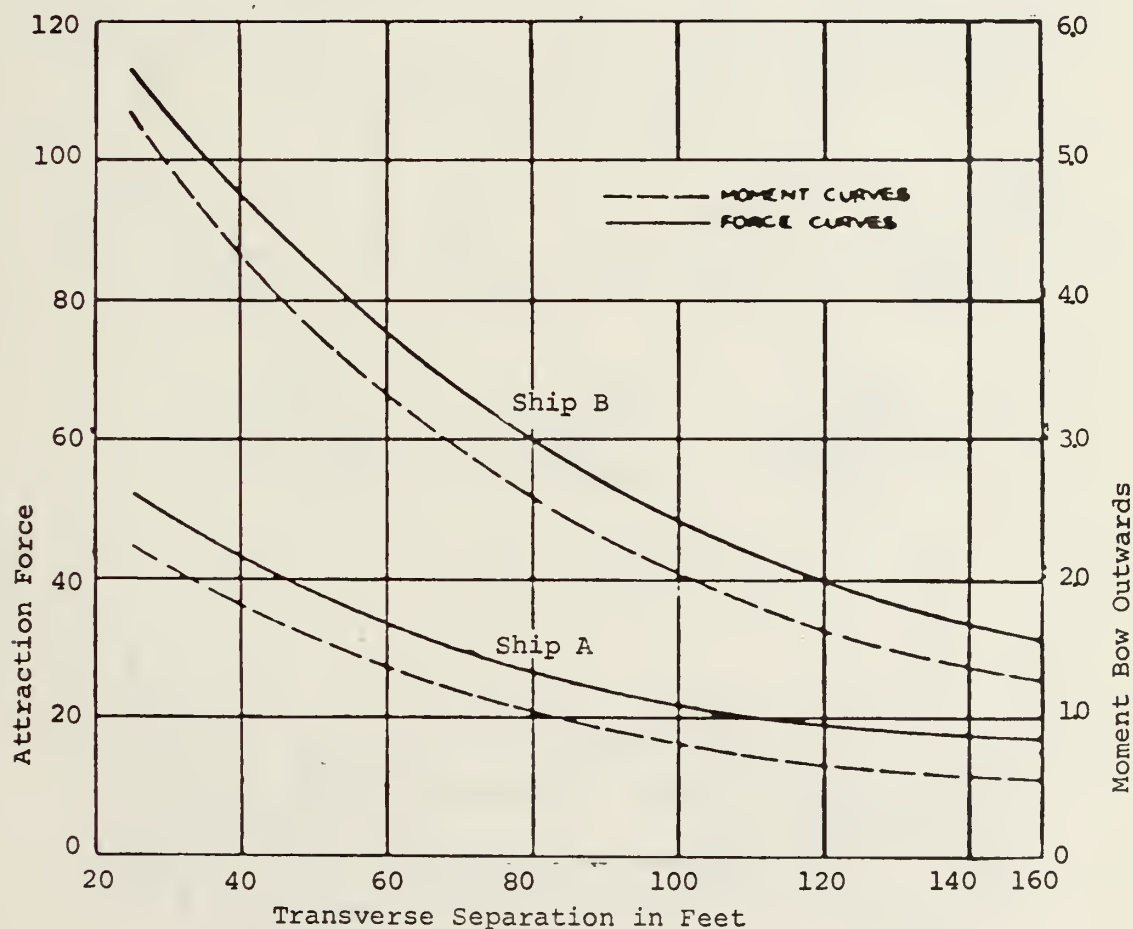


Figure 2. Variation of Interaction Forces and Moments with Transverse Separation.

Figure 3 shows the measured interaction forces and moments, and correcting rudder angles for the case of 100-foot separation.

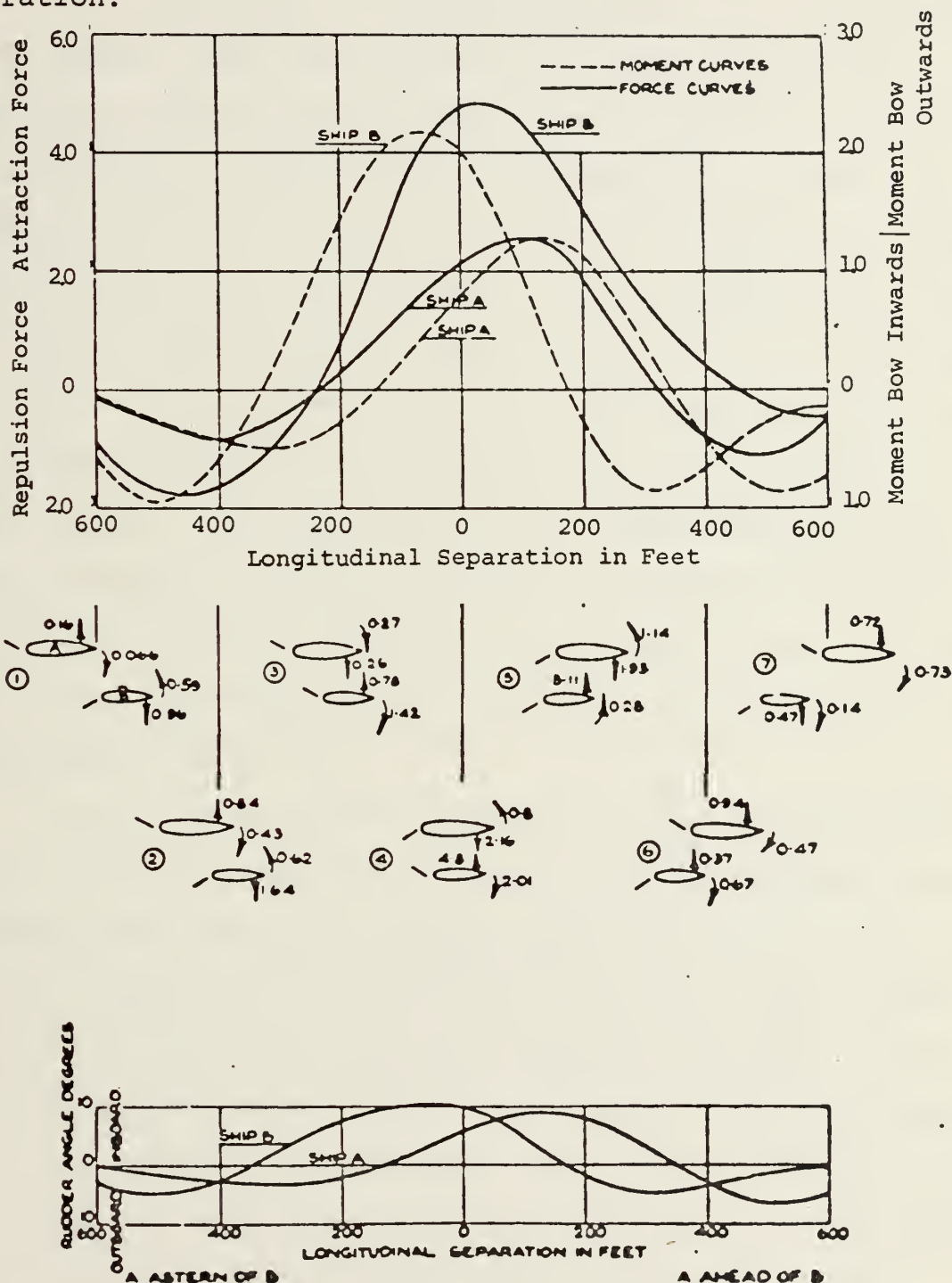


Figure 3. Measured Interaction Forces and Moments and Correcting Rudder Angles

One hundred foot separation beam-to-beam.

It is evident from Figs. 1 and 3 that there are positions when both the interaction force and moment tend to draw one ship toward the other. Such positions are 3 for ship A and 5 for ship B. In these positions the rudder deflection angles are such that the rudder moments oppose the interaction moments. However with these deflection angles the rudder force tends to add to the force of attraction. Therefore in these positions (3 and 5), it is necessary to deflect the rudder sufficiently so that not only the interaction moment is overcome, but also a yaw angle is introduced that creates an outboard force that counteracts both the attraction and the rudder force. By these means the two ships should be able to avoid collision in positions 3 and 5, provided there is enough transverse separation between the two ships so that the available rudder force can correct the inward swing caused by the interaction moment.

It should be noted that position 3 immediately precedes, and position 5 immediately follows the directly abeam position when the two ships have to apply opposite rudder to keep on parallel courses. Thus in the short space of time between positions 3 and 4 for ship A and between positions 4 and 5 for ship B, the rudder has to swing from a large port deflection to a starboard deflection. Obviously the precise timing when this has to be done is not easy to choose. It is therefore true that the two ships suffer the greatest risk of collision in positions 3 and 5, which would be augmented if the seas were rough and a heavy wind were blowing.

III. EQUATIONS OF MOTION

A. THE GENERAL CASE

It is clear that bodies moving in a fluid medium are free to move with six degrees of freedom. In order to define the equations of motion, a right-hand rectangular coordinate system is established, the origin of which is chosen to be in the body itself, as shown in Fig. 4. The origin could very well be at the center of gravity but for generalizing the problem, it can be placed anywhere else. The origin and the axes are fixed with respect to the body but movable with respect to the axes fixed in space. It is assumed that at $t=0$ the two systems coincide

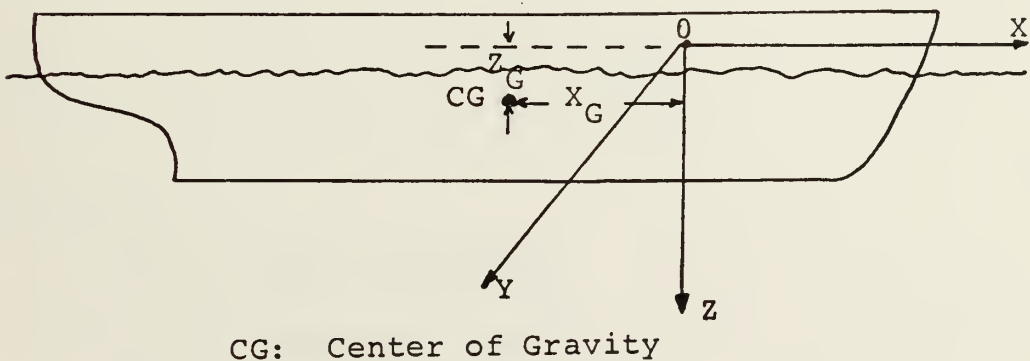


Figure 4. The Coordinates System.

The motion of a rigid body is expressed by Newton's Laws of motion in vector form:

$$\vec{F}(\text{External Force}) = \frac{d}{dt} [\text{Momentum}] \quad (3.1)$$

$$\vec{M}(\text{External Moment}) = \frac{d}{dt} [\text{Angular Momentum}]$$

Then the six equations (six degrees of freedom) describing the ship's motion have been found [4] to be:

$$\begin{aligned} X &= m[\dot{U}-RV+QW-X_G(R^2+Q^2) + Y_G(PQ-\dot{R}) + Z_G(PR+\dot{Q})] \\ Y &= m[\dot{V}-PW+RU+X_G(\dot{R}+PW) - Y_G(P^2+R^2) + Z_G(PQ-\dot{P})] \\ Z &= m[\dot{W}-QU+PV+X_G(PR-\dot{Q}) + Y_G(\dot{P}+QR) - Z_G(Q^2+P^2)] \\ L &= \dot{P}I_X + (I_Z-I_Y)QR + m[Y_G(\dot{W}-QU+PV) - Z_G(\dot{V}-PW+RU)] \\ M &= \dot{Q}I_Y + (I_X-I_Z)PR + m[Z_G(\dot{U}-RV+QW) - X_G(\dot{W}-QU+PV)] \\ N &= \dot{R}I_Z + (I_Y-I_X)PQ + m[X_G(\dot{V}-PW+RU) - Y_G(\dot{U}-RV+QW)] \end{aligned} \quad (3.2)$$

satisfying equations --

$$\begin{aligned} \vec{F} &= \vec{i} \cdot X + \vec{j} \cdot Y + \vec{k} \cdot Z \\ \vec{M} &= \vec{i} \cdot L + \vec{j} \cdot M + \vec{k} \cdot N \end{aligned} \quad (3.3)$$

and where

- m - mass of the ship
- X, Y, Z - components of force in the X, Y, Z directions
- L, M, N - components of applied moment about the X, Y, Z axes
- U, V, W - components of velocity in the X, Y, Z directions
- X_G, Y_G, Z_G - distances of origin from center of gravity in the X, Y, Z directions
- P, Q, R - components of angular velocity about the X, Y, Z axes
- I_X, I_Y, I_Z - moments of inertia about the X, Y, Z axes

Equations (3.2) describe the reaction of the rigid body to applied forces as a function of the geometric and physical characteristics of the body itself. They do not include any of the applied external forces such as propeller thrust, or rudder forces, or forces and moments due to the fins (if they exist). They do not include the reaction forces of the fluid (hydrodynamic forces) nor do they include the waves and wind forces. The reactions of the rigid body (ship) are characteristic and must be considered in studying control problems.

B. THE HORIZONTAL PLANE MOTION

It is clear that the ship's motion in calm waters is described only by the following three equations:

$$\begin{aligned} X &= m[\dot{U}-RV+QW-X_G(R^2+Q^2) + Y_G(PQ-\dot{R}) + Z_G(PR+\dot{Q})] \text{ (surge)} \\ Y &= m[\dot{V}+UR-PW+X_G(\dot{R}+PW) - Y_G(P^2+R^2) + Z_G(RQ-\dot{P})] \text{ (sway)} \\ N &= \dot{R}I_z + (I_y-I_x)PQ + m[X_G(\dot{V}-PW+RU) - Y_G(\dot{U}-RV+QW)] \text{ (yaw)} \end{aligned} \quad (3.4)$$

Under the assumption of calm water conditions it is true that Roll=Pitch=Heave = 0, and the three equations of motion are the so-called horizontal plane equations. Therefore since horizontal plane motion implies: $P=\dot{P}=Q=\dot{Q}=W=\dot{W}=0$ equations (3.4) can be written as follows:

$$\begin{aligned} X &= m[\dot{U}-RV-X_GR^2-Y_G\dot{R}] \\ Y &= m[\dot{V}+UR+X_G\dot{R}-Y_GR^2] \\ N &= \dot{R}I_z+m[X_G(\dot{V}+RU)-Y_G(\dot{U}-RV)] \end{aligned} \quad (3.5)$$

Assuming that the coordinate's origin is placed on the center of gravity of the ship, then $X_G = Y_G = 0$ and Eqs. (3.5) become:

$$\begin{aligned} X &= m[\dot{U} - RV] \\ Y &= m[\dot{V} + UR] \\ N &= \dot{R}I_z \end{aligned} \quad (3.6)$$

Defining:

$$\Psi_T = \text{Yaw angle}$$

and substituting the relations $R = \dot{\Psi}_T$ and $\dot{R} = \ddot{\Psi}_T$ into (3.6) yields:

$$\begin{aligned} X &= m[\dot{U} - \dot{\Psi}_T V] \\ Y &= m[\dot{V} + U\dot{\Psi}_T] \\ N &= \ddot{\Psi}_T I_z \end{aligned} \quad (3.7)$$

1. Linearization of the Horizontal Plane Equations

An equilibrium condition is one of a steady forward motion of the ship for which $U_0 = \text{Constant}$, $V_0 = 0$, $\Psi_0 = \text{Constant}$. Then considering small perturbations u, v, ψ ;
 $U = U_0 + u$

$$V = V_0 + v$$

$$\Psi_T = \Psi_0 + \psi$$

$$R = R_0 + r$$

The right-hand parts of Eqs. (3.7) take the form:

$$\dot{V} + U\dot{\Psi}_T = \dot{V}_0 + \dot{v} + (U_0 + u)(\dot{\Psi}_0 + \dot{\psi})$$

But $\dot{V}_0 = \dot{\Psi}_0 = 0$, and dropping second-order terms yields:

$$\dot{V} + U\dot{\Psi}_T = \dot{v} + U_0\dot{\psi}$$

And since $V_0 = 0$, Eqs. (3.7) becomes:

$$\begin{aligned} X &= m\dot{u} \\ Y &= m[\dot{v} + \Psi U_0] \\ N &= I_z \ddot{\Psi} \end{aligned} \quad (3.8)$$

Then setting again $\dot{\Psi} = r$ and $\ddot{\Psi} = \dot{r}$ gives

$$\begin{aligned} X &= m\dot{u} \\ Y &= m[\dot{v} + r \cdot U_0] \\ N &= I_z \dot{r} \end{aligned} \quad (3.9)$$

The hydrodynamic forces and moment for these particular motions have been found [5].

$$\begin{aligned} X &= \frac{\partial X}{\partial u} \cdot \Delta u + \frac{\partial X}{\partial \dot{u}} \cdot \dot{u} = X_u \cdot \Delta u + X_{\dot{u}} \cdot \dot{u} \\ Y &= \frac{\partial Y}{\partial v} \cdot v + \frac{\partial Y}{\partial \dot{v}} \dot{v} + \frac{\partial Y}{\partial r} \cdot r + \frac{\partial Y}{\partial \dot{r}} \dot{r} = Y_v \cdot v + Y_{\dot{v}} \cdot \dot{v} + Y_r \cdot r + Y_{\dot{r}} \cdot \dot{r} \\ N &= \frac{\partial N}{\partial v} \cdot v + \frac{\partial N}{\partial \dot{v}} \dot{v} + \frac{\partial N}{\partial r} \cdot r + \frac{\partial N}{\partial \dot{r}} \dot{r} = N_v \cdot v + N_{\dot{v}} \cdot \dot{v} + N_r \cdot r + N_{\dot{r}} \cdot \dot{r} \end{aligned} \quad (3.10)$$

The cross-coupled derivatives Y_r , $Y_{\dot{r}}$, N_v , $N_{\dot{v}}$, even though they have small nonzero values, have to be included unless the ship under consideration is symmetrical about the YZ-plane, which is not usually the case. Thus taking into account the hydrodynamic derivatives Eqs. (3.9) become:

$$\begin{aligned} 0 &= -X_u(u - U_0) + (m - X_{\dot{u}})\dot{u} \\ 0 &= -Y_v \cdot v + (m - Y_{\dot{v}})\dot{v} - (Y_r - mU_0)r - Y_{\dot{r}} \cdot \dot{r} \\ 0 &= -N_v \cdot v - N_{\dot{v}} \cdot \dot{v} - N_r \cdot r + (I_z - N_{\dot{r}})\dot{r} \end{aligned} \quad (3.11)$$

2. Nondimensionalization of the Horizontal Plane Equations

For several reasons and mainly for computer simulation purposes, it is necessary to nondimensionalize equations (3.11). Table I gives dimensionalized and nondimensionalized quantities and their conversion relations. The prime symbol indicates the nondimensional form of each quantity.

In equations (3.11) the force equations are divided through by $\frac{\rho}{2} L^2 V^2$ and the moment equation by $\frac{\rho}{2} L^3 V^2$ yielding

$$\begin{aligned} 0 &= -Y'_V \cdot v' + (m' - Y'_V) \dot{v}' - (Y'_R - m' \cdot U'_0) r' - Y'_R \cdot \dot{r}' \\ 0 &= -N'_V \cdot v' - N'_V \cdot \dot{v}' - N'_R \cdot r' + (I'_Z - N'_R) \dot{r}' \\ 0 &= -X'_u (u' - U'_0) + (m' - X'_u) \dot{u}' \end{aligned} \quad (3.12)$$

If the motion of the ship is to be considered under external perturbations and with controls working, equations (3.12) must include terms expressing forces and moments due to sea and wind excitations (in rough weather), and forces and moments caused by rudder deflections or movable fin deflections. The rudder and fins forces and moments are considered control elements. The rest of the forces and moments are not normally controlled inputs, but they must be included in cases where the ship has to be controlled in their presence. In this last category, belong the interactive forces and moments generated in the cases of ships in close underway replenishment stations.

Considering only rudder control inputs, equations (3.12) become

TABLE I
NONDIMENSIONALIZATION RELATIONS

m	m'	$m' = m/\rho \frac{L^3}{2}$
u	u'	$u' = u/V$
v	v'	$v' = v/V$
\dot{u}	\dot{u}'	$\dot{u}' = \dot{u}L/V^2$
\dot{v}	\dot{v}'	$\dot{v}' = \dot{v}L/V^2$
I_z	I_z'	$I_z' = m/\rho \frac{L^5}{2}$
r	r'	$r' = rL/V$
\dot{r}	\dot{r}'	$\dot{r}' = \dot{r}L^2/V^2$
X_u	X_u'	$X_u' = X_u/\rho \frac{L^2V}{2}$
$X_{\dot{u}}$	$X_{\dot{u}}'$	$X_{\dot{u}}' = X_{\dot{u}}/\rho \frac{L^3}{2}$
Y_v	Y_v'	$Y_v' = Y_v/\rho \frac{L^2V}{2}$
Y_r	Y_r'	$Y_r' = Y_r/\rho \frac{L^3V}{2}$
$Y_{\dot{v}}$	$Y_{\dot{v}}'$	$Y_{\dot{v}}' = Y_{\dot{v}}/\rho \frac{L^3}{2}$
$Y_{\dot{r}}$	$Y_{\dot{r}}'$	$Y_{\dot{r}}' = Y_{\dot{r}}/\rho \frac{L^4}{2}$
N_v	N_v'	$N_v' = N_v/\rho \frac{L^3V}{2}$
$N_{\dot{v}}$	$N_{\dot{v}}'$	$N_{\dot{v}}' = N_{\dot{v}}/\rho \frac{L^4}{2}$
N_r	N_r'	$N_r' = N_r/\rho \frac{L^4V}{2}$
$N_{\dot{r}}$	$N_{\dot{r}}'$	$N_{\dot{r}}' = N_{\dot{r}}/\rho \frac{L^5}{2}$
$N_v \cdot v$	$N_v' \cdot v'$	$N_v' \cdot v' = N_v \cdot v/\rho \frac{L^3V^2}{2}$
$N_{\dot{v}} \cdot \dot{v}$	$N_{\dot{v}}' \cdot \dot{v}'$	$N_{\dot{v}}' \cdot \dot{v}' = N_{\dot{v}} \cdot \dot{v}/\rho \frac{L^3V^2}{2}$

ρ = Sea water density in lb/ft³

L = Length of ship in ft.

V = Speed of ship in ft/sec.

$$\begin{aligned}
-Y'_V \cdot v' + (m' - Y'_V) \dot{v}' - (Y'_R - m' U'_0) r' - Y'_R \dot{r}' &= Y'_\delta \cdot \delta \quad (\text{Sway}) \\
-N'_V \cdot v' - N'_V \dot{v}' - N'_R \cdot r' + (I'_Z - N'_R) \dot{r}' &= N'_\delta \cdot \delta \quad (\text{Yaw}) \\
-X'_u \cdot (u' - U'_0) + (m' - X'_u) \dot{u}' &= X'_\delta \cdot \delta \quad (\text{Surge})
\end{aligned}
\tag{3.13}$$

where, δ = Rudder-deflection angle, measured from the XZ-plane of the ship to the plane of the rudder; positive deflection corresponds to a turn to port for rudder(s) located at stern.

$Y'_\delta, N'_\delta, X'_\delta$ = nondimensionalized forms of the rudder forces and moment,

$Y_\delta = \frac{\partial Y}{\partial \delta}, N_\delta = \frac{\partial N}{\partial \delta}, X_\delta = \frac{\partial X}{\partial \delta}$, respectively.

3. Manipulation of the Equations for Computer Simulation

Equations (3.13) can be written as follows:

$$\begin{aligned}
v'(s) [s(m' - Y'_V) - Y'_V] + r'(s) [-sY'_R + (m'U'_0 - Y'_R)] &= Y'_\delta \cdot \delta(s) \\
v'(s) [-s \cdot N'_V - N'_V] + r'(s) [s(I'_Z - N'_R) - N'_R] &= N'_\delta \cdot \delta(s) \\
u'(s) [s(m' - X'_u)] + u'(s) [-X'_u] + X'_u \cdot \frac{U'_0}{s} &= X'_\delta \cdot \delta(s)
\end{aligned}
\tag{3.14}$$

or

$$\begin{aligned}
\frac{v'(s)}{s} [s^2(m' - Y'_V) - sY'_V] + \psi'(s) [-s^2Y'_R + s(m'U'_0 - Y'_R)] &= Y'_\delta \cdot \delta(s) \\
\frac{v'(s)}{s} [-s^2N'_V - sN'_V] + \psi'(s) [s^2(I'_Z - N'_R) - sN'_R] &= N'_\delta \cdot \delta(s) \\
\frac{u'(s)}{s} [s^2(m' - X'_u) - sX'_u] + X'_u \cdot \frac{U'_0}{s} &= X'_\delta \cdot \delta(s)
\end{aligned}
\tag{3.15}$$

or

$$\frac{v'(s)}{s} [\alpha_{aA}s^2 + \beta_{aA}s + \gamma_{aA}] + \psi'(s) [\alpha_{bA}s^2 + \beta_{bA}s + \gamma_{bA}] = Y'_\delta \cdot \delta(s)$$

$$\frac{v'(s)}{s} [\alpha_{aB}s^2 + \beta_{aB}s + \gamma_{aB}] + \psi'(s) [\alpha_{bB}s^2 + \beta_{bB}s + \gamma_{bB}] = N'_\delta \cdot \delta(s)$$

$$\frac{u'(s)}{s} [\alpha_{cC}s^2 + \beta_{cC}s + \gamma_{cC}] = X'_\delta \cdot \delta(s) - X'_u \cdot \frac{U'_0}{s}$$

(3.16)

where

$$\alpha_{aA} = m' - Y'_v$$

$$\beta_{aA} = -Y'_v$$

$$\gamma_{aA} = 0$$

$$\alpha_{bA} = Y'_r$$

$$\beta_{bA} = m' \cdot U'_0 - Y'_r$$

$$\gamma_{bA} = 0$$

$$\alpha_{aB} = N'_v$$

$$\beta_{aB} = -N'_v$$

$$\gamma_{aB} = 0$$

$$\alpha_{bB} = (I'_z - N'_r)$$

$$\beta_{bB} = -N'_r$$

$$\gamma_{bB} = 0$$

$$\alpha_{cC} = m' - X'_u$$

$$\beta_{cC} = -X'_u$$

$$\gamma_{cC} = 0$$

Setting

$$\frac{v'(s)}{s} = A(s) \quad \text{or} \quad v' = \dot{A}$$

$$\psi'(s) = B(s) \quad \text{or} \quad \psi = B$$

$$\frac{u'(s)}{s} = C(s) \quad \text{or} \quad u' = \dot{C}$$

and

$$IF1 = Y\delta' \cdot \delta(s) = KA1 \cdot D1$$

$$IF2 = N\delta' \cdot \delta(s) = KB1 \cdot D1$$

$$IF3 = X\delta' \cdot \delta(s) - X'_u \cdot \frac{U'_0}{s} = KC1 \cdot D1 - X'_u \cdot U'_0$$

equations (3.16) can be written as follows:

$$\begin{aligned} \alpha_{aA} \ddot{A} + \beta_{aA} \dot{A} + \gamma_{aA} A + \alpha_{bA} \ddot{B} + \beta_{bA} \dot{B} + \gamma_{bA} B &= IF1 \\ \alpha_{aB} \ddot{A} + \beta_{aB} \dot{A} + \gamma_{aB} A + \alpha_{bB} \ddot{B} + \beta_{bB} \dot{B} + \gamma_{bB} B &= IF2 \\ \alpha_{cC} \ddot{C} + \beta_{cC} \dot{C} + \gamma_{cC} C &= IF3 \end{aligned} \quad (3.17)$$

or

$$\begin{aligned} \alpha_{aA} \ddot{A} + \alpha_{bA} \ddot{B} &= I1 \\ \alpha_{aB} \ddot{A} + \alpha_{bB} \ddot{B} &= I2 \\ \alpha_{cC} \ddot{C} &= I3 \end{aligned} \quad (3.18)$$

where

$$\begin{aligned} I1 &= -\beta_{aA} \dot{A} - \gamma_{aA} A - \beta_{bA} \dot{B} - \gamma_{bA} B + IF1 \\ I2 &= -\beta_{aB} \dot{A} - \gamma_{aB} A - \beta_{bB} \dot{B} - \gamma_{bB} B + IF2 \\ I3 &= -\beta_{cC} \dot{C} - \gamma_{cC} C + IF3 \end{aligned}$$

and solving for \ddot{A} , \ddot{B} , \ddot{C} , yields

$$\ddot{A} = \frac{\begin{vmatrix} I_1 & \alpha_{bA} & 0 \\ I_2 & \alpha_{bB} & 0 \\ I_3 & 0 & \alpha_{cC} \end{vmatrix}}{[\Delta]}, \quad \ddot{B} = \frac{\begin{vmatrix} \alpha_{aA} & I_1 & 0 \\ \alpha_{aB} & I_2 & 0 \\ 0 & I_3 & \alpha_{cC} \end{vmatrix}}{[\Delta]}, \quad \ddot{C} = \frac{\begin{vmatrix} \alpha_{aA} & \alpha_{bA} & I_1 \\ \alpha_{aB} & \alpha_{bB} & I_2 \\ 0 & 0 & I_3 \end{vmatrix}}{[\Delta]} \quad (3.19a)$$

where

$$\Delta = \begin{vmatrix} \alpha_{aA} & \alpha_{bA} & 0 \\ \alpha_{aB} & \alpha_{bB} & 0 \\ 0 & 0 & \alpha_{cC} \end{vmatrix}$$

and furthermore solving for v' , u' , ψ' yields

$$\begin{aligned} v' &= \dot{A} = v'_0 + \int \ddot{A} \, dt \\ \psi' &= \psi_0 + \int \dot{B} \, dt = \psi_0 + \int [\dot{B}_0 + \int \ddot{B} dt] dt \\ u' &= \dot{C} = u'_0 + \int \ddot{C} \, dt \end{aligned} \quad (3.19b)$$

Then according to Figure 5 [4]:

$$\begin{aligned} \dot{y}' &= u' \sin \psi' + v' \cos \psi' \\ \dot{x}' &= u' \cos \psi' - v' \sin \psi' \end{aligned} \quad (3.19c)$$

Thus giving

$$y' = y'_0 + \int \dot{y}' dt \quad \text{and} \quad x' = x'_0 + \int \dot{x}' dt \quad (3.19d)$$

Equations (3.19) form the basis of CSMP computer program I.

This program was simulated for a ship in motion with a speed

$U'_0 = \frac{U_0}{V} = 1.0$ and a constant rudder deflection $\delta = \Delta l = 0.1$

rad. The results are shown in Figures 6 and 7. Figure 6 is

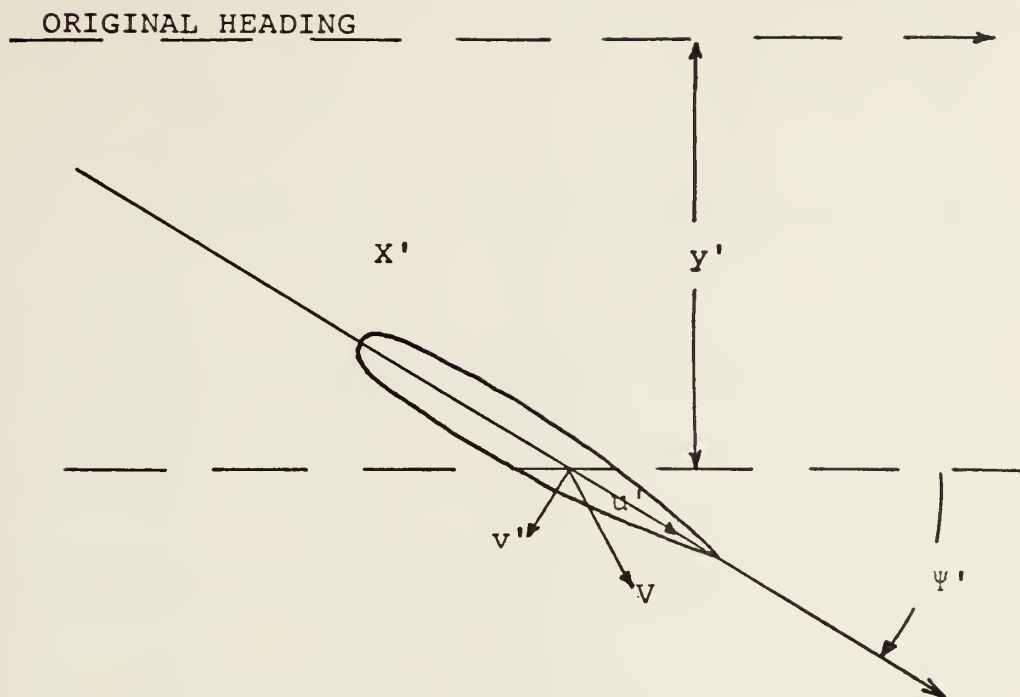


Figure 5. The Horizontal Plane Motions.

a Yaw versus Time plot, and Figure 7 is the turning radius (sway versus surge) characteristic of the ship.

This ship is a class D surface ship and its hydrodynamic coefficients¹ given in Table II were used in the computer simulation, and will be considered in all further work throughout this thesis.

4. Stability Investigation

The test for stability is to establish an equilibrium situation and determine whether the system returns to the

¹ Courtesy of Dr. Grant Hagen, David Taylor Model Basin.

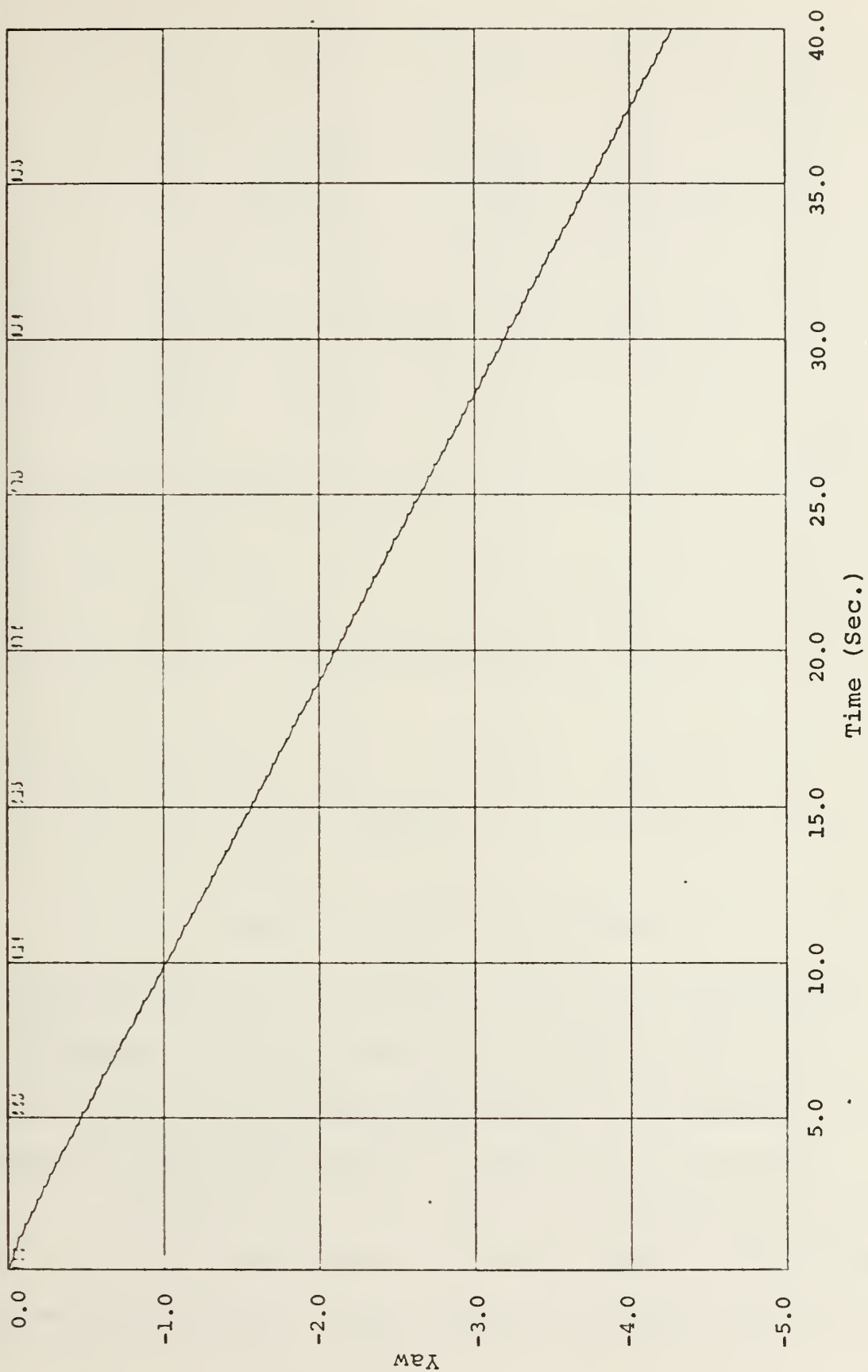


Figure 6. Yaw vs. Time

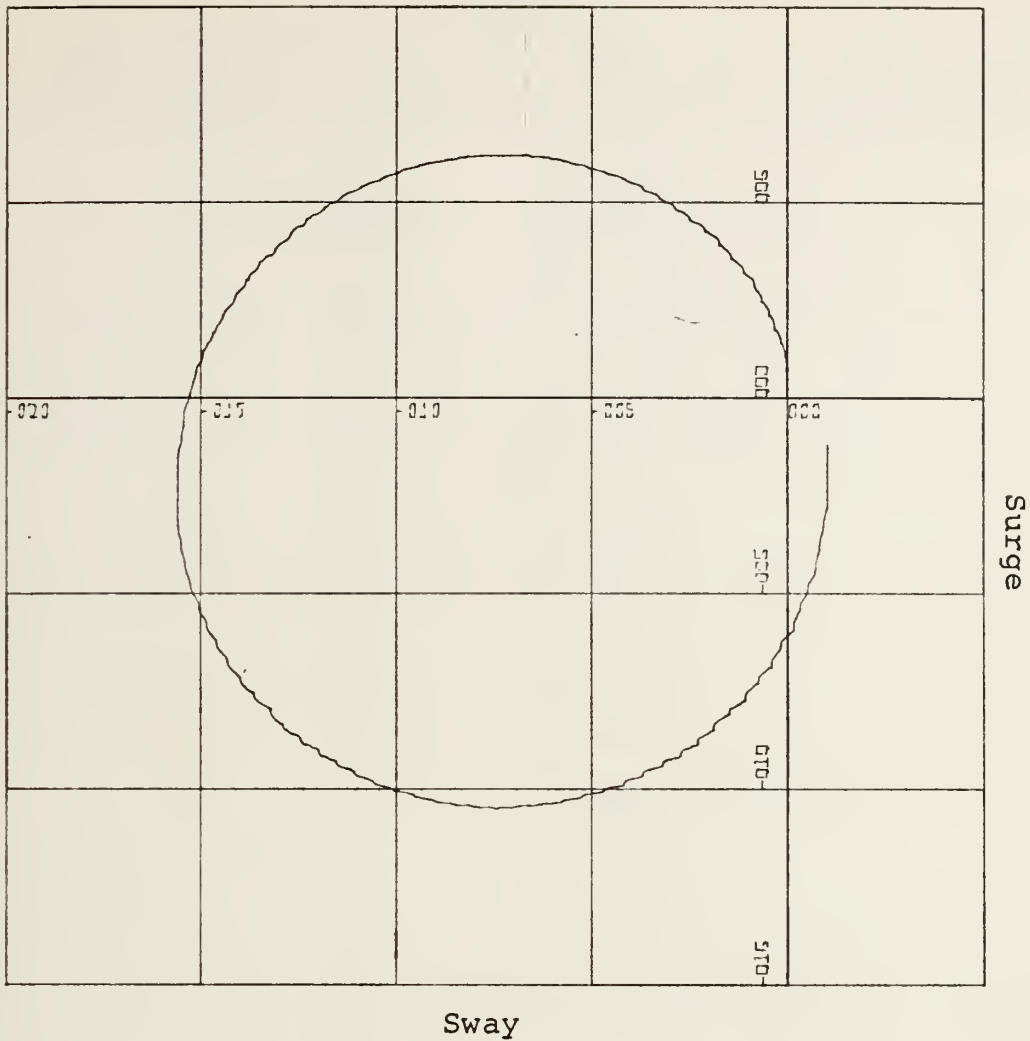


Figure 7. The Turning Radius.

original condition of equilibrium after an infinitesimal disturbance. If it returns to the original equilibrium condition, after the disturbance is removed, it is stable. If it departs or has the tendency to depart, it is unstable.

For the ship, such an equilibrium condition is the one of a straight ahead motion at constant speed. A ship which is dynamically unstable, cannot maintain straight line motion

TABLE II

NONDIMENSIONALIZED COEFFICIENTS FOR THE CLASS D SHIP

m'	0.0045
I_z'	0.0003
N_r'	-0.0012
N_r^{\cdot}	-0.0002
N_v'	-0.0012
N_v^{\cdot}	-0.0001
N_δ'	-0.00084
Y_r'	0.004
Y_r^{\cdot}	-0.0002
Y_v'	-0.0063
Y_v^{\cdot}	-0.0025
Y_δ'	0.0019
X_u'	-0.0019
X_u^{\cdot}	-0.0036
X_δ'	-0.0011

when there is no rudder deflection. The behavior of the ship (plant) is to be examined under the influence of forcing functions (disturbances), with controls (rudders) fixed ($\delta = 0$), or under the influence of controls acting as disturbances.

With controls fixed and for $U_0' = 1.0$ Eqs. (3.16)

become:

$$\begin{aligned}\frac{v'(s)}{s} [\alpha_{aA}s^2 + \beta_{aB}s] + \psi'(s) [\alpha_{bA}s^2 + \beta_{bA}s] &= 0 \\ \frac{v'(s)}{s} [\alpha_{aB}s^2 + \beta_{aB}s] + \psi'(s) [\alpha_{bB}s^2 + \beta_{bB}s] &= 0 \\ \frac{u'(s)}{s} [\alpha_{cC}s^2 + \beta_{cC}s] &= -Xu'\end{aligned}\quad (3.20)$$

Since a steady forward motion, $\Delta u' = 0$, has been assumed, then the surge equation can be neglected giving:

$$\begin{aligned}\frac{v'(s)}{s} [\alpha_{aA}s^2 + \beta_{aB}s] + \psi'(s) [\alpha_{bA}s^2 + \beta_{bA}s] &= 0 \\ \frac{v'(s)}{s} [\alpha_{aB}s^2 + \beta_{aB}s] + \psi'(s) [\alpha_{bB}s^2 + \beta_{bB}s] &= 0\end{aligned}\quad (3.21)$$

or

$$\begin{aligned}v'(s) [\alpha_{aA}s + \beta_{aA}] + r'(s) [\alpha_{bA}s + \beta_{bA}] &= 0 \\ v'(s) [\alpha_{aB}s + \beta_{aB}] + r'(s) [\alpha_{bB}s + \beta_{bB}] &= 0\end{aligned}\quad (3.22)$$

The characteristic equation is:

$$[(\alpha_{aA} \alpha_{bB} - \alpha_{aB} \alpha_{bA})s^2 + (\alpha_{bB} \beta_{aA} + \alpha_{aA} \beta_{bB} - \alpha_{aB} \beta_{bA} - \alpha_{bA} \beta_{aB})s + (\beta_{aA} \beta_{bB} - \beta_{aB} \beta_{bA})] = 0$$

or

$$c_1 s^2 + c_2 s + c_3 = 0 \quad (3.23)$$

where

$$\begin{aligned}c_1 &= \alpha_{aA} \alpha_{bB} - \alpha_{aB} \alpha_{bA} \\ c_2 &= \alpha_{bB} \beta_{aA} + \alpha_{aA} \beta_{bB} - \alpha_{aB} \beta_{bA} - \alpha_{bA} \beta_{aB} \\ c_3 &= \beta_{aA} \beta_{bB} - \beta_{aB} \beta_{bA}\end{aligned}$$

The roots of the characteristic equation are

$$\rho_{1,2} = \frac{-c_2 \pm \sqrt{c_2^2 - 4c_1c_3}}{2c_1} = \frac{-c_2/c_1 \pm \left[\left[c_2/c_1 \right]^2 - 4c_3/c_1 \right]^{1/2}}{2}$$

For stability it is necessary that $\rho_1 < 0$ and $\rho_2 < 0$, which is guaranteed when

$$\beta = \frac{c_2}{c_1} > 0 \quad \text{and} \quad \gamma = \frac{c_3}{c_1} > 0$$

That is the condition for stability is given as:

$$\beta > 0 \quad \text{and} \quad \gamma > 0. \quad (3.24)$$

For the class D surface ship implementing values of Table II gives:

$$\begin{aligned} \alpha_{aA} &= 0.007 \\ \beta_{aA} &= 0.0063 \\ \alpha_{bA} &= 0.0002 \\ \beta_{bA} &= 0.0005 \\ \alpha_{aB} &= 0.0001 \\ \beta_{aB} &= 0.0012 \\ \alpha_{bB} &= 0.0005 \\ \beta_{bB} &= 0.0012 \\ \alpha_{cC} &= 0.00486 \\ \beta_{cC} &= 0.0012 \end{aligned}$$

And further substitution gives:

$$\beta = 3.227 > 0, \quad \gamma = 2.0 > 0$$

It is therefore proved that this particular ship possesses controls fixed stability.

5. The Transfer Functions

For $U_0' = 1.0$ Eqs. (3.16) can be written again:

$$\begin{aligned} \frac{v'(s)}{s} \cdot K_{aA} + \psi'(s) \cdot K_{bA} + \frac{u'(s)}{s} \cdot (0) &= Y_\delta' \cdot \delta(s) \\ \frac{v'(s)}{s} \cdot K_{aB} + \psi'(s) \cdot K_{bB} + \frac{u'(s)}{s} \cdot (0) &= N_\delta' \cdot \delta(s) \end{aligned} \quad (3.25)$$

$$\frac{v'(s)}{s} \cdot 0 + \psi'(s) (0) + \frac{u'(s)}{s} \cdot K_{cC} = X_\delta' \cdot \delta(s) - Xu' \cdot \frac{1}{s}$$

where

$$K_{aA} = \alpha_{aA} s^2 + \beta_{aA} s$$

$$K_{bA} = \alpha_{bA} s^2 + \beta_{bA} s$$

$$K_{aB} = \alpha_{aB} s^2 + \beta_{aB} s$$

$$K_{bB} = \alpha_{bB} s^2 + \beta_{bB} s$$

$$K_{cC} = \alpha_{cC} s^2 + \beta_{cC} s$$

Then solving Eqs. (3.25) yields

$$\begin{aligned} \frac{v'(s)}{s} / \delta(s) &= \frac{\begin{vmatrix} Y_\delta' & K_{bA} & 0 \\ N_\delta' & K_{bB} & 0 \\ X_\delta' & 0 & K_{cC} \end{vmatrix}}{[\Delta]} = \frac{[N_1]}{[\Delta]}, \quad \psi'(s) / \delta(s) = \frac{\begin{vmatrix} K_{aA} & Y_\delta' & 0 \\ K_{aB} & N_\delta' & 0 \\ 0 & X_\delta' & K_{cC} \end{vmatrix}}{[\Delta]} = \frac{[N_2]}{[\Delta]}, \\ \frac{u'(s)}{s} / \delta(s) &= \frac{\begin{vmatrix} K_{aA} & K_{bA} & Y_\delta' \\ K_{aB} & K_{bB} & N_\delta' \\ 0 & 0 & X_\delta' \end{vmatrix}}{[\Delta]} = \frac{[N_3]}{[\Delta]} \end{aligned} \quad (3.26)$$

where

$$\Delta = \begin{vmatrix} K_{aA} & K_{bA} & 0 \\ K_{aB} & K_{bB} & 0 \\ 0 & 0 & K_{cC} \end{vmatrix}$$

or

$$\Delta = K_{aA} \cdot K_{bB} K_{cC} - K_{aB} K_{bA} K_{cC} = K_{cC} \cdot [K_{aA} K_{bB} - K_{aB} K_{bA}]$$

or

$$\Delta = s(\alpha_{cC} s + \beta_{cC}) [s(\alpha_{aA} s + \beta_{aA}) s(\alpha_{bB} s + \beta_{bB}) - s(\alpha_{aB} s + \beta_{aB}) s(\alpha_{bA} s + \beta_{bA})]$$

and finally

$$\Delta = s^3(\alpha_{cC} s + \beta_{cC}) [\alpha_{aA} \cdot \alpha_{bB} - \alpha_{aB} \alpha_{bA}] s^2 + (\alpha_{aA} \beta_{bB} + \alpha_{bB} \beta_{aA} - \alpha_{aB} \beta_{bA} - \beta_{aB} \alpha_{bA}) s + (\beta_{aA} \beta_{bB} - \beta_{aB} \beta_{bA})]$$

Evaluating N_1 , N_2 , and N_3 and substituting into Eqs. (3.26) yields:

$$\frac{v'(s)}{\delta(s)} = \frac{K_v(s+z_v)}{s^2 + \beta s + \gamma} \quad (3.27)$$

$$\frac{\psi'(s)}{\delta(s)} = \frac{K_r(s+z_r)}{s(s^2 + \beta s + \gamma)} \quad (3.28)$$

$$\frac{u'(s)}{\delta(s)} = \frac{K_u}{s+p_u} \quad (3.29)$$

where

$$\begin{aligned}
 K_v &= \frac{Y'_\delta \alpha_{bB} - N'_\delta \alpha_{bA}}{\alpha_{aA} \alpha_{bB} - \alpha_{aB} \alpha_{bA}} \\
 Z_v &= \frac{Y'_\delta \beta_{bB} - N'_\delta \beta_{bA}}{Y'_\delta \alpha_{bB} - N'_\delta \alpha_{bA}} \\
 K_r &= \frac{N'_\delta \alpha_{aA} - Y'_\delta \alpha_{aB}}{\alpha_{aA} \alpha_{bB} - \alpha_{aB} \alpha_{bA}} \\
 Z_r &= \frac{N'_\delta \beta_{aA} - Y'_\delta \beta_{aB}}{N'_\delta \alpha_{aA} - Y'_\delta \alpha_{aB}} \\
 K_u &= \frac{X\delta'}{\alpha_{cC}} \\
 P_u &= \frac{\beta_{cC}}{\alpha_{cC}} \\
 \beta &= \frac{\alpha_{aB} \beta_{bB} + \alpha_{bB} \beta_{aA} - \alpha_{aB} \beta_{bA} - \beta_{aB} \alpha_{bA}}{\alpha_{aA} \alpha_{bB} - \alpha_{aB} \alpha_{bA}} \\
 \gamma &= \frac{\beta_{aA} \beta_{bB} - \beta_{aB} \beta_{bA}}{\alpha_{aA} \alpha_{bB} - \alpha_{aB} \alpha_{bA}}
 \end{aligned} \tag{3.30}$$

The transfer function determination leads now to the block diagram representation of the plant (ship) shown in Fig. 8, where the rudder deflection is taken as the only control input.

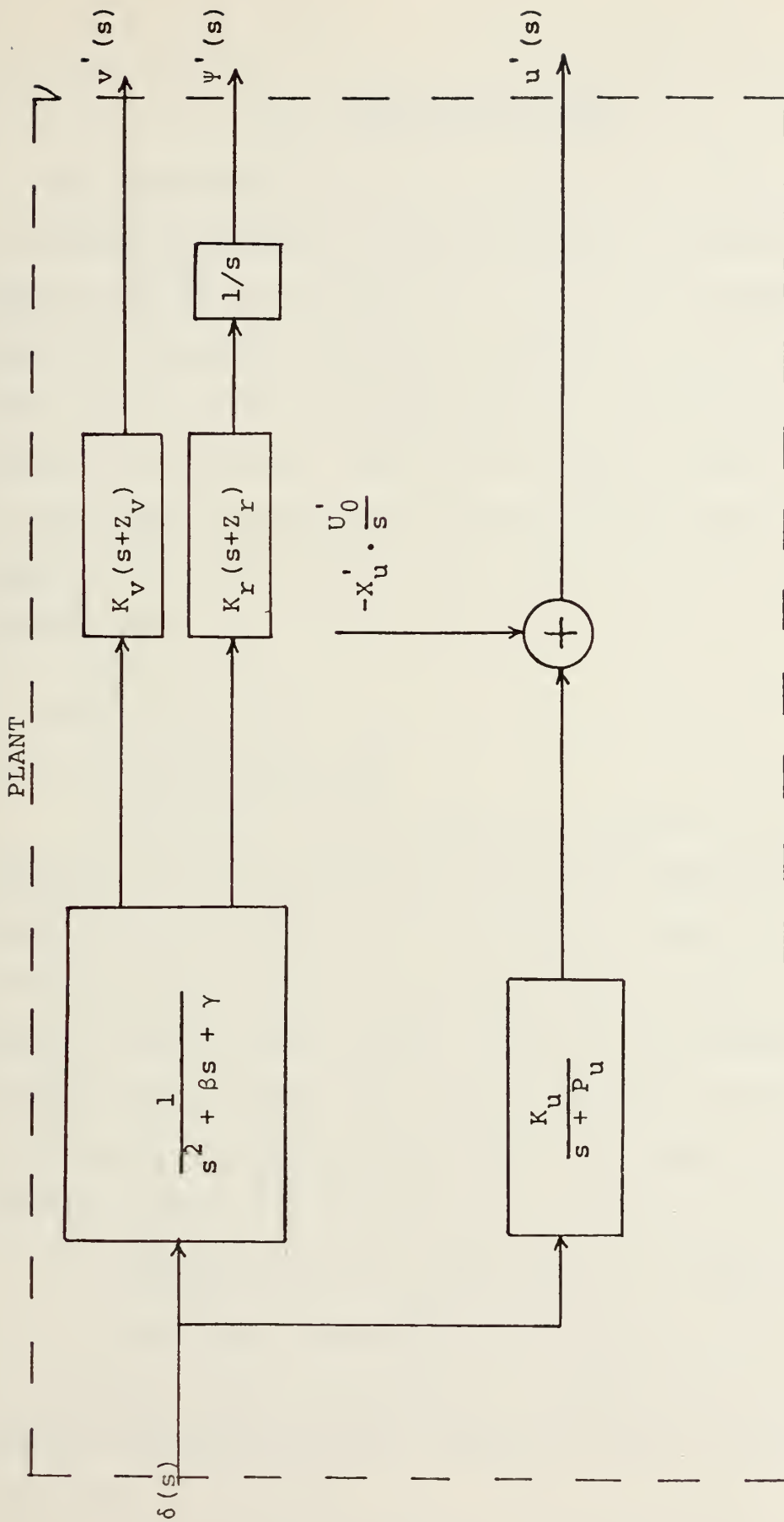


Figure 8. The Open Loop System

IV. THE CONTROL LOOPS

The replenishment at sea operation as a control problem is directly dependent on the operational responsibilities of each of the ships involved in it. It has already been stated that the replenishing ship is responsible for course keeping only, and that the receiving ship is responsible for course and station keeping. Hence from a control point of view it is clear that two different control loops should be required, namely, the course keeping loop and the distance (station) keeping loop.

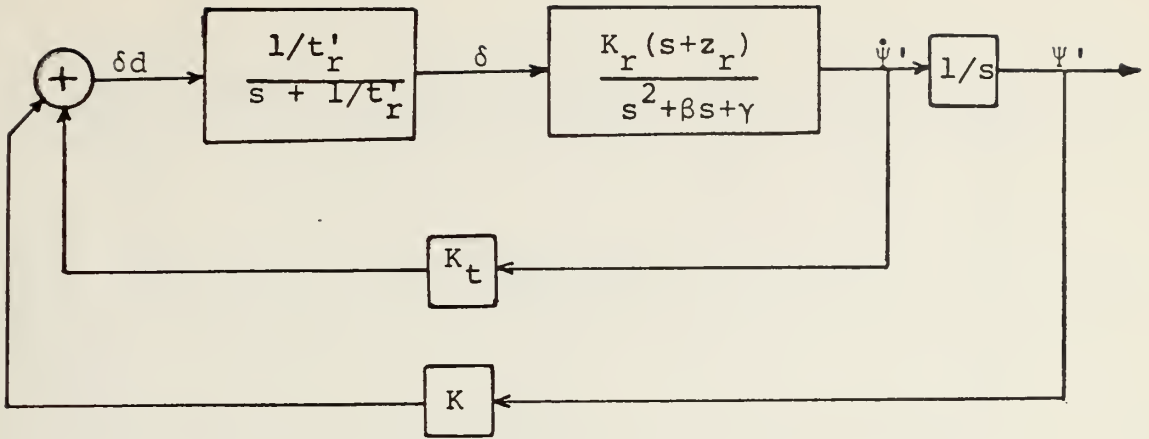
A. PATHKEEPING LOOP

1. The Block Diagram

The control loop is shown in Figure 9. For a course keeping action the important output variables of the control system should be the yaw, ψ' , and the yaw rate, $\dot{\psi}'$. Since the main objective is course keeping and not course changing, the control loop should include no input yaw reference. It includes the existing time lag t_r between the helm's action and the desired actual displacement of the rudder itself. Non-dimensionalized, the time lag t_r' is usually taken equal to 0.1.

2. Determination of K and K_t for a Desired Performance.

The proper values for K and K_t are to be determined for a critically damped response of the loop and therefore for a desired performance of the system dictated by the condition:
 $\zeta = 1.0$.



δ_d - Ordered Rudder angle (helm)

δ - Actual Rudder angle

t'_r - Nondimensional time lag (≈ 0.1)

Figure 9. The Course Keeping Loop.

A computer program called PARAM "A" implemented accordingly will yield a set of pairs of values of K and K_t for any desired ζ for a range of operating frequencies.

The equivalent of the controlled loop of Fig. 9 is shown in Fig. 10. Then comparison of Figs. 10 and 11 gives:

$$G = \frac{10K_r(s+z_r)}{s(s+10)(s^2+\beta s+\gamma)}, \quad H = K + K_t \cdot s$$

The overall closed loop transfer function is

$$\frac{G}{1-GH} = \frac{\frac{10K_r(s+z_r)}{s(s+10)(s^2+\beta s+\gamma)}}{1 - \frac{10K_r(s+z_r)(K_t s + K)}{s(s+10)(s^2+\beta s+\gamma)}}$$

and the characteristic equation is

$$s(s+10)(s^2+\beta s+\gamma) - 10K_r(s+z_r)(K_t s + K) = 0$$

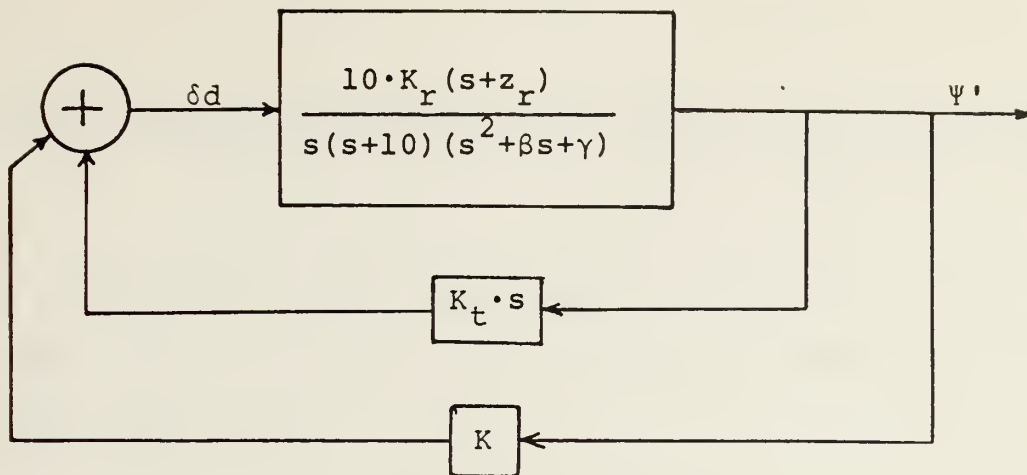


Figure 10. The Equivalent Loop.

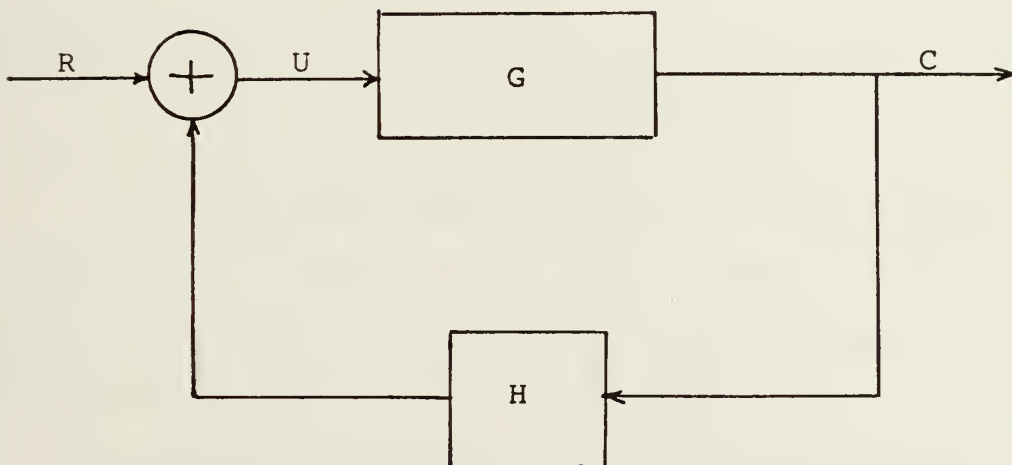


Figure 11. General Feedback Control Loop.

Or

$$s^4 + (10 + \beta)s^3 + [(10\beta + \gamma) - 10K_r K_t]s^2 + [10\gamma - 10K_r K - 10K_r Z_r K_t]s - 10K_r Z_r K = 0 \quad (4.1)$$

Considering again the Class D surface ship and evaluating Eqs. (3.30) according to the values of Table II for the required quantities

$$K_r = -1.74425$$

$$Z_r = 1.24744$$

$$\beta = 3.227$$

$$\gamma = 2.0$$

And now Eq. (4.1) becomes:

$$s^4 + 13.2227s^3 + [34.27 - 17.4425K_t]s^2 + [20.0 + 17.4425K + 21.7584K_t]s + 21.7584K = 0 \quad (4.2)$$

The aforementioned PARAM "A" computer program was simulated for the case of Eq. (4.2) and for $\zeta = 1.0$ and a sample of the computer result containing the permissible pairs of values of K and K_t is given in Table III.

3. Computer Simulation of the Controlled Plant

From Fig. 9 the relation between δ and δd is

$$\delta = \frac{10}{s+10} \delta d \quad \text{and} \quad \delta d = (K_t s + K) \Psi'$$

Therefore:

$$\delta = \frac{10}{s+10} (K_t s + K) \Psi' \quad (4.3)$$

TABLE III
APPROPRIATE VALUES OF PARAMETERS K AND K_t

K	K_t	Ω	ζ	Third Parameter
0.80689E-01	-0.38959E00	0.29719E00	0.10000E01	0.13665E08
0.84989E-01	-0.37530E00	0.30447E00	0.10000E01	0.13665E08
0.89534E-01	-0.36055E00	0.31193E00	0.10000E01	0.13665E08
0.94340E-01	-0.34533E00	0.31957E00	0.10000E01	0.13665E08
0.99424E-01	-0.32961E00	0.32740E00	0.10000E01	0.13665E08
0.10481E00	-0.31338E00	0.33542E00	0.10000E01	0.13665E08
0.11050E00	-0.29659E00	0.34364E00	0.10000E01	0.13665E08
0.11654E00	-0.27924E00	0.35206E00	0.10000E01	0.13665E08
0.12294E00	-0.26129E00	0.36058E00	0.10000E01	0.13665E08
0.12973E00	-0.24270E00	0.36952E00	0.10000E01	0.13665E08
0.13693E00	-0.22344E00	0.37857E00	0.10000E01	0.13665E08
0.14458E00	-0.20343E00	0.38785E00	0.10000E01	0.13665E08
0.15271E00	-0.18277E00	0.39735E00	0.10000E01	0.13665E08
0.16136E00	-0.16127E00	0.40709E00	0.10000E01	0.13665E08
0.17055E00	-0.13893E00	0.41706E00	0.10000E01	0.13665E08
0.18037E00	-0.11569E00	0.42728E00	0.10000E01	0.13665E08
0.19084E00	-0.91504E-01	0.43775E00	0.10000E01	0.13665E08
0.20201E00	-0.66294E-01	0.44847E00	0.10000E01	0.13665E08
0.21395E00	-0.39952E-01	0.45946E00	0.10000E01	0.13665E08
0.22673E00	-0.12515E-01	0.47072E00	0.10000E01	0.13665E08
0.24042E00	-0.16227E-01	0.48225E00	0.10000E01	0.13665E08
0.25512E00	-0.46335E-01	0.49406E00	0.10000E01	0.13665E08
0.27092E00	-0.77921E-01	0.50617E00	0.10000E01	0.13665E08
0.28793E00	-0.11112E00	0.51857E00	0.10000E01	0.13665E08
0.306281E00	-0.14608E00	0.53127E00	0.10000E01	0.13665E08
0.32611E00	-0.18295E00	0.54422E00	0.10000E01	0.13665E08
0.34759E00	-0.22194E00	0.55762E00	0.10000E01	0.13665E08
0.37092E00	-0.26326E00	0.57129E00	0.10000E01	0.13665E08
0.39630E00	-0.30716E00	0.58528E00	0.10000E01	0.13665E08
0.42435E00	-0.35390E00	0.59962E00	0.10000E01	0.13665E08
0.45438E00	-0.40339E00	0.61431E00	0.10000E01	0.13665E08
0.48768E00	-0.45745E00	0.62933E00	0.10000E01	0.13665E08
0.52442E00	-0.51515E00	0.64478E00	0.10000E01	0.13665E08
0.56503E00	-0.57744E00	0.66058E00	0.10000E01	0.13665E08
0.61027E00	-0.64502E00	0.67676E00	0.10000E01	0.13665E08
0.66074E00	-0.71869E00	0.69333E00	0.10000E01	0.13665E08
0.71740E00	-0.79934E00	0.71033E00	0.10000E01	0.13665E08
0.78137E00	-0.88335E00	0.72773E00	0.10000E01	0.13665E08
0.85405E00	-0.98700E00	0.74556E00	0.10000E01	0.13665E08
0.93718E00	-0.10571E01	0.76383E00	0.10000E01	0.13665E08

Substitution of Eq. (4.3) into (3.15) gives, for $U'_0 = 1.0$:

$$\begin{aligned}\frac{v'(s)}{s} [s^2 (m' - Y'_V) - sY'_V] + \psi'(s) [-s^2 Y'_r + s(m' - Y'_r)] &= \frac{10Y'_\delta (K_t s + K) \psi'}{s+10} \\ \frac{v'(s)}{s} [-s^2 N'_V - sN'_V] + \psi'(s) [s^2 (I'_Z - N'_r) - sN'_r] &= \frac{10N'_\delta (K_t s + K) \psi'}{s+10} \\ \frac{u'(s)}{s} [s^2 (m' - X'_u) - sX'_u] + X'_u &= \frac{10X'_\delta (K_t s + K) \psi'}{s+10}\end{aligned}\quad (4.4)$$

And after manipulations Eqs. (4.4) becomes

$$\begin{aligned}\frac{v'(s)}{s} [s^3 (m' - Y'_V) + \{10(m' - Y'_V) - Y'_V\} s^2 + (-10Y'_V) s] + \psi'(s) [-Y'_r s^3 + \{(m' - Y'_r) \\ - 10Y'_r\} s^2 + \{10(m' - Y'_r) - 10Y'_\delta K_t\} s - 10Y'_\delta K] &= 0 \\ \frac{v'(s)}{s} [-N'_V s^3 - (N'_V + 10N'_V) s^2 - 10N'_V s] + \psi'(s) [s^3 (I'_Z - N'_r) + \{10(I'_Z - N'_r) - N'_r\} s^2 \\ - (10N'_r + 10N'_\delta K_t) s - 10N'_\delta K] &= 0 \\ \frac{u'(s)}{s} [s^3 (m' - X'_u) + \{10(m' - X'_u) - X'_u\} s^2 - 10X'_u s] + \psi'(s) [-10X'_\delta K_t s - 10X'_\delta K] \\ &= -10X'_u\end{aligned}\quad (4.5)$$

or

$$\begin{aligned}\frac{v'(s)}{s} [\alpha_{aA} s^3 + \beta_{aA} s^2 + \gamma_{aA} s + \delta_{aA}] + \psi'(s) [\alpha_{aA} s^3 + \beta_{bA} s^2 + \gamma_{bA} s + \delta_{bA}] &= 0 \\ \frac{v'(s)}{s} [\alpha_{aB} s^3 + \beta_{aB} s^2 + \gamma_{aB} s + \delta_{aB}] + \psi'(s) [\alpha_{bB} s^3 + \beta_{bB} s^2 + \gamma_{bB} s + \delta_{bB}] &= 0 \\ \frac{u'(s)}{s} [\alpha_{cC} s^3 + \beta_{cC} s^2 + \gamma_{cC} s + \delta_{cC}] + \psi'(s) [\alpha_{bC} s^3 + \beta_{bC} s^2 + \gamma_{bC} s + \delta_{bC}] &= -10X'_u\end{aligned}\quad (4.6)$$

where

$$\begin{aligned}
 \alpha_{aA} &= (m' - Y_v') \\
 \beta_{aA} &= 10(m' - Y_v') - Y_v' \\
 \gamma_{aA} &= -10Y_v' \\
 \delta_{bA} &= 0 \\
 \alpha_{bA} &= -Y_r' \\
 \beta_{bA} &= (m' - Y_r') - 10Y_r' \\
 \gamma_{bA} &= 10(m' - Y_r') - 10Y_\delta' K_t \\
 \delta_{bA} &= -10Y_\delta' K \\
 \alpha_{aB} &= -N_v' \\
 \beta_{aB} &= -N_v' - 10N_v' \\
 \gamma_{aB} &= -10N_v' \\
 \delta_{aB} &= 0 \\
 \alpha_{bB} &= (I_z' - N_r') \\
 \beta_{bB} &= 10(I_z' - N_r') - N_r' \\
 \gamma_{bB} &= -10N_r' - 10N_\delta' \cdot K_t \\
 \delta_{bB} &= -10N_\delta' K \\
 \alpha_{bC} &= 0 \\
 \beta_{bC} &= 0 \\
 \gamma_{bC} &= -10X_\delta' K_t \\
 \delta_{bC} &= -10X_\delta' K \\
 \alpha_{cC} &= (m' - X_u') \\
 \beta_{cC} &= 10(m' - X_u') - X_u' \\
 \gamma_{cC} &= -10X_u' \\
 \delta_{cC} &= 0
 \end{aligned}$$

and setting

$$\frac{v'(s)}{s} = A(s) \quad \text{or} \quad v'(t) = \dot{A}$$

$$\Psi'(s) = B(s) \quad \text{or} \quad \Psi'(t) = B$$

$$\frac{u'(s)}{s} = C(s) \quad \text{or} \quad u'(t) = \dot{C}$$

gives

$$\alpha_{aA} \ddot{A} + \beta_{aA} \ddot{A} + \gamma_{aA} \dot{A} + \delta_{aA} A + \alpha_{bA} \ddot{B} + \beta_{bA} \ddot{B} + \gamma_{bA} \dot{B} + \delta_{bA} B = 0$$

$$\alpha_{aB} \ddot{A} + \beta_{aB} \ddot{A} + \gamma_{aB} \dot{A} + \delta_{aB} A + \alpha_{bB} \ddot{B} + \beta_{bB} \ddot{B} + \gamma_{bB} \dot{B} + \delta_{bB} B = 0 \quad (4.8)$$

$$\alpha_{cC} \ddot{C} + \beta_{cC} \ddot{C} + \gamma_{cC} \dot{C} + \delta_{cC} C + \alpha_{bC} \ddot{B} + \beta_{bC} \ddot{B} + \gamma_{bC} \dot{B} + \delta_{bC} B = IF3$$

or

$$\alpha_{aA} \ddot{A} + \alpha_{bA} \ddot{B} = I1$$

$$\alpha_{aB} \ddot{A} + \alpha_{bB} \ddot{B} = I2 \quad (4.9)$$

$$\alpha_{cC} \ddot{C} + \alpha_{bC} \ddot{B} = I3$$

where

$$IF3 = -10X_u'$$

$$I1 = -\beta_{aA} \ddot{A} - \gamma_{aA} \dot{A} - \delta_{aA} A - \beta_{bA} \ddot{B} - \gamma_{bA} \dot{B} - \delta_{bA} B$$

$$I2 = -\beta_{aB} \ddot{A} - \gamma_{aB} \dot{A} - \delta_{aB} A - \beta_{bB} \ddot{B} - \gamma_{bB} \dot{B} - \delta_{bB} B \quad (4.10)$$

$$I3 = -\beta_{cC} \ddot{C} - \gamma_{cC} \dot{C} - \delta_{cC} C - \beta_{bC} \ddot{B} - \gamma_{bC} \dot{B} - \delta_{bC} B$$

Now the computer program I can be modified accordingly to include the control action, and the resulting computer program II was implemented and simulated for the Class D surface ship, and for an arbitrary pair of values of $K = 0.78137$ and $K_t = 0.88835$ from Table III, corresponding to a

critically damped ($\zeta=1.0$) response, for the following course keeping case:

The ship is originally on a straightforward motion ($U'_0 = 1.0$) at a specified course. Due to external disturbances (waves, wind) it acquires a certain yaw, $\Psi = 0.2$ and a certain yaw rate, $\dot{\Psi} = 0.1$. The results of the control action to bring and keep the ship back to the original course are shown in Fig. 12.

The existing steady state error can be considered negligible and furthermore justified since the simulation, as it was carried out, included the effect of the coupling among the three equations of motion, whereas this affect was neglected in the procedure for optimization of K and K_t .

B. THE DISTANCE KEEPING LOOP

1. A Special Case

The feedback control loop in a block diagram form is shown in Fig. 13. Here a somewhat unrealistic but helpful assumption is made, that the control's objective is to keep the lateral motion (sway) of one individual ship to zero, with respect to its original path, or to the set of axes placed in space. This assumption will be relaxed later in the general case of two ships involved in the replenishment at sea operation, where an initial lateral separation between the ships is established.

Therefore the loop is now a zero input reference one, and the main important output variables are the sway, y , and the sway rate, \dot{y} .

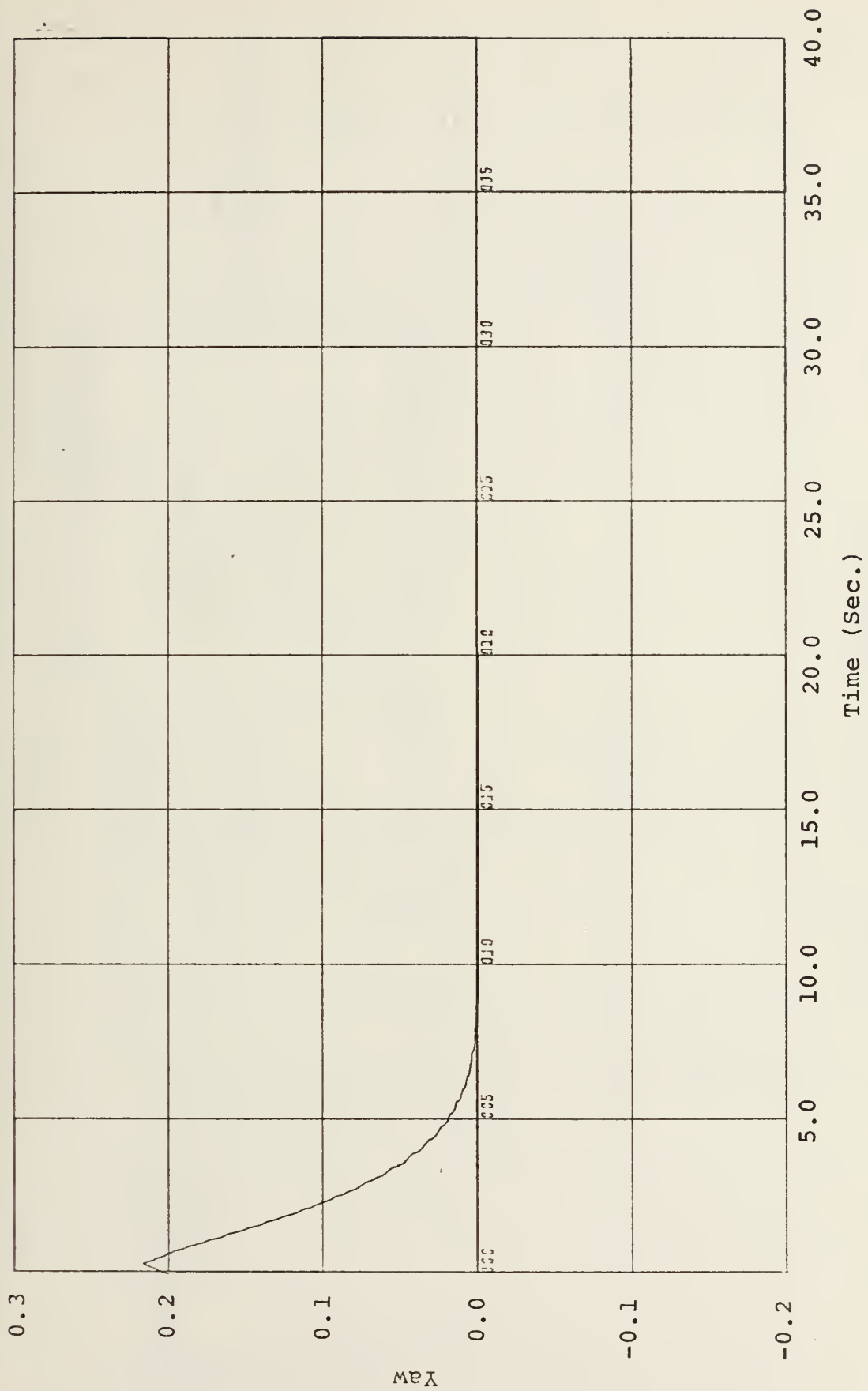


Figure 12. The Course Keeping System's Response

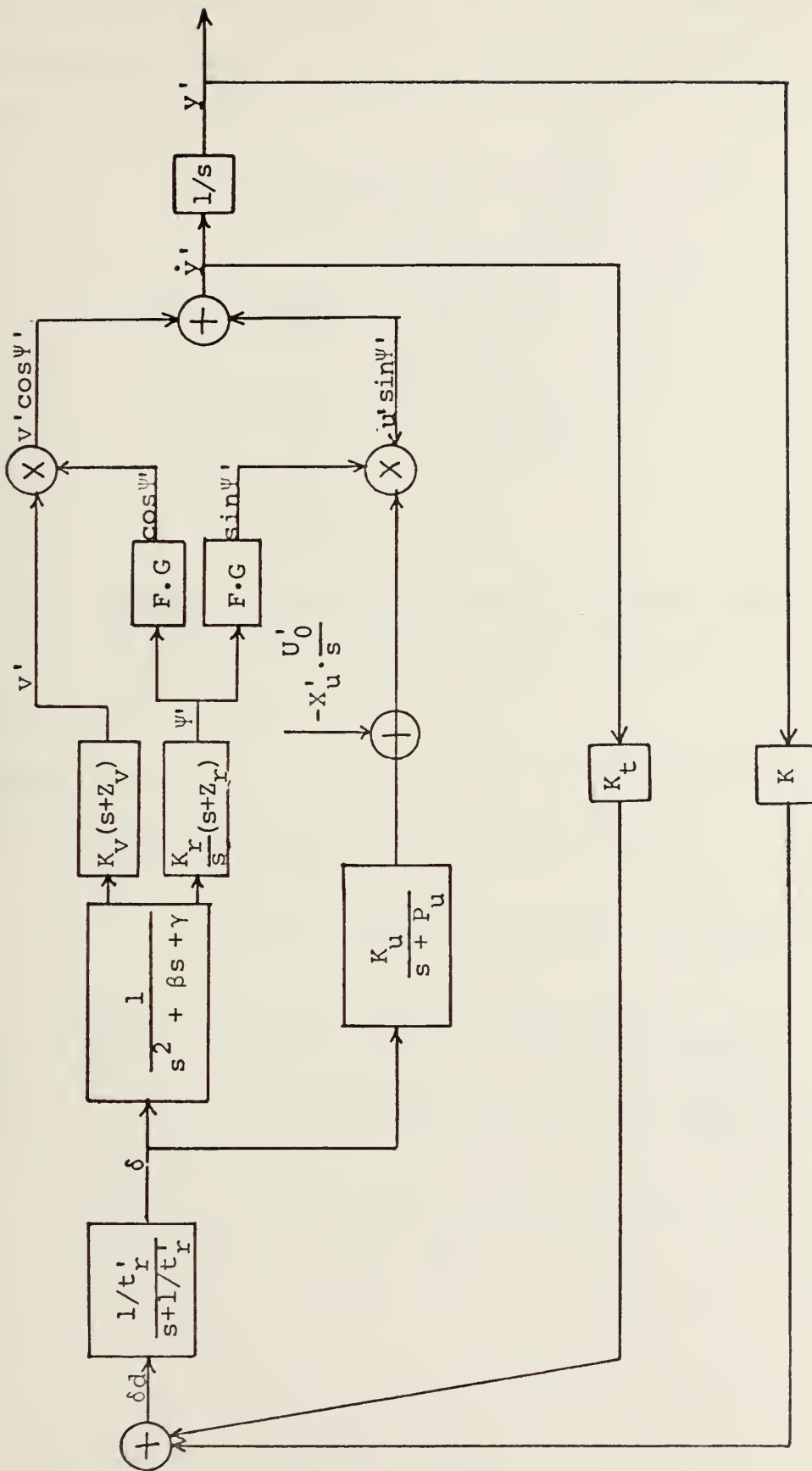


Figure 13. The Distance Keeping Loop.

The equations derived from Fig. 5, and the transfer functions on page 32 are repeated here for further explanation of Fig. 13.

$$\begin{aligned}\dot{y}' &= u' \sin \psi' + v' \cos \psi' \\ \dot{x} &= u' \cos \psi' - v' \sin \psi'\end{aligned}$$

$$\frac{v'(s)}{\delta(s)} = \frac{K_v(s+Z_v)}{s^2+\beta s+\gamma} \quad (3.27)$$

$$\frac{\psi'(s)}{\delta(s)} = \frac{K_r(s+Z_r)}{s(s^2+\beta s+\gamma)} \quad (3.28)$$

$$\frac{u'(s)}{\delta(s)} = \frac{K_u}{s+P_u} \quad (3.29)$$

2. Computer Program for the Controlled Plant

The relation between δ and δd is again $\delta = \frac{10}{s+10} \delta d$ (4.11)

and $\delta d = (K_t s + K) y'$.

Substitution of Eq. (4.11) into (3.15) for $U_0' = 1.0$ gives

$$\begin{aligned}\frac{v'(s)}{s} [\alpha_{aA} s^3 + (\beta_{aA} + 10\alpha_{aA}) s^2 + 10\beta_{aA} s + 10\gamma_{aA}] \\ + \psi'(s) [\alpha_{bA} s^3 + (\beta_{bA} + 10\alpha_{bA}) s^2 + 10\beta_{bA} s + 10\gamma_{bA}] = IF1\end{aligned}$$

$$\begin{aligned}\frac{v'(s)}{s} [\alpha_{aB} s^3 + (\beta_{aB} + 10\alpha_{aB}) s^2 + 10\beta_{aB} s + 10\gamma_{aB}] \\ + \psi'(s) [\alpha_{bB} s^3 + (\beta_{bB} + 10\alpha_{bB}) s^2 + 10\beta_{bB} s + 10\gamma_{bB}] = IF2\end{aligned}$$

$$\frac{u'(s)}{s} [\alpha_{cC} s^3 + (\beta_{cC} + 10\alpha_{cC}) s^2 + 10\beta_{cC} s + 10\gamma_{cC}] = IF3 \quad (4.12)$$

where

$$IF1 = 10Y'_\delta \cdot \delta d = 10KA1 \cdot \delta d$$

$$IF2 = 10N'_\delta \cdot \delta d = 10KB1 \cdot \delta d$$

$$IF3 = 10X'_\delta \cdot \delta d - 10X'_u = 10KC1 \cdot \delta d - 10X'_u$$

and setting

$$\frac{v'(s)}{s} = A(s) \quad \text{or} \quad v'(t) = \dot{A}$$

$$\psi'(s) = B(s) \quad \text{or} \quad \psi'(t) = B$$

$$\frac{u'(s)}{s} = C(s) \quad \text{or} \quad u'(s) = \dot{C}$$

gives

$$\alpha'_{aA} \ddot{A} + \beta'_{aA} \ddot{A} + \gamma'_{aA} \dot{A} + \alpha'_{bA} \ddot{B} + \beta'_{bA} \ddot{B} + \gamma'_{bA} \dot{B} = IF1$$

$$\alpha'_{aB} \ddot{A} + \beta'_{aB} \ddot{A} + \gamma'_{aB} \dot{A} + \alpha'_{bB} \ddot{B} + \beta'_{bB} \ddot{B} + \gamma'_{bB} \dot{B} = IF2 \quad (4.13)$$

$$\alpha'_{cC} \ddot{C} + \beta'_{cC} \ddot{C} + \gamma'_{cC} \dot{C} = IF3$$

$$\text{or} \quad \alpha'_{aA} \ddot{A} + \alpha'_{bA} \ddot{B} = I1$$

$$\alpha'_{aB} \ddot{A} + \alpha'_{bB} \ddot{B} = I2$$

$$\alpha'_{cC} \ddot{C} = I3$$

where

$$I1 = IF1 - \beta'_{aA} \ddot{A} - \gamma'_{aA} \dot{A} - \beta'_{bA} \ddot{B} - \gamma'_{bA} \dot{B}$$

$$I2 = IF2 - \beta'_{aB} \ddot{A} - \gamma'_{aB} \dot{A} - \beta'_{bB} \ddot{B} - \gamma'_{bB} \dot{B}$$

$$I3 = IF3$$

(4.14)

and

$$\alpha'_{aA} = \alpha_{aA}$$

$$\beta'_{aA} = \beta_{aA} + 10\alpha_{aA}$$

$$\gamma'_{aA} = 10\beta_{aA}$$

$$\alpha'_{bA} = \alpha_{bA}$$

$$\beta'_{bA} = \beta_{bA} + 10\alpha_{bA}$$

$$\gamma'_{bA} = 10\beta_{bA}$$

$$\alpha'_{aB} = \alpha_{aB}$$

$$\beta'_{aB} = 10\alpha_{aB} + \beta_{aB}$$

$$\gamma'_{aB} = 10\beta_{aB}$$

$$\alpha'_{bB} = \alpha_{bB}$$

$$\beta'_{bB} = \beta_{bB} + 10\alpha_{bB}$$

$$\gamma'_{bB} = 10\beta_{bB}$$

$$\alpha'_{cC} = \alpha_{cC}$$

$$\beta'_{cC} = \beta_{cC} + 10\alpha_{cC}$$

$$\gamma'_{cC} = 10\beta_{cC}$$

(4.14a)

Now the computer program I is modified to reflect the appropriate control action, and this results to computer program III.

3. Determination of K and K_t for a Desired Performance

The proper values for K and K_t are again to be determined for the system's response under the condition: $\zeta = 1.0$.

For small angles of yaw it is true that $\sin \Psi' \approx \Psi'$ and $\cos \Psi' \approx 1.0$, and therefore the Eq. $y' = u' \sin \Psi' + v' \cos \Psi'$ can be written as $y' \approx \Psi' + v'$. (4.15)

From Eqs. (3.27 and 3.28) respectively:

$$\begin{aligned}\Psi' &= \frac{K_r(s+Z_r)}{s(s^2+\beta s+\gamma)} \delta(s) \\ v' &= \frac{K_v(s+Z_v)}{s^2+\beta s+\gamma} \delta(s)\end{aligned}$$

and making use of Eq. (4.11) gives

$$\begin{aligned}\Psi' &= \frac{10K_r(s+Z_r) \cdot \delta d}{s(s+10)(s^2+\beta s+\gamma)} \\ v' &= \frac{10K_v(s+Z_v) \delta d}{(s+10)(s^2+\beta s+\gamma)}\end{aligned}$$

Therefore Eq. (4.15) becomes:

$$\dot{y}'/\delta d = \frac{10K_r(s+Z_r) + 10sK_v(s+Z_v)}{s(s+10)(s^2+\beta s+\gamma)}$$

and

$$y'/\delta d = \frac{10K_r(s+Z_r) + 10sK_v(s+Z_v)}{s^2(s+10)(s^2+\beta s+\gamma)} \quad (4.16)$$

But from Fig. 13:

$$\delta d = (K_t s + K) y' \quad (4.17)$$

Now based on Eqs. (4.16 and 4.17) the equivalent of the control loop of Fig. 13 is given in Fig. 14 in agreement with the general form of Fig. 11. Therefore the closed loop transfer function is now:

$$\frac{G}{1-GH} = \frac{10K_r(s+Z_r)+10s K_v(s+Z_v)}{s^2(s+10)(s^2+\beta s+\gamma)-(K_t s+K)[10K_r(s+Z_r)+10sK_v(s+Z_v)]} \quad (4.18)$$

and the characteristic equations is

$$s^5+(10+\beta)s^4+(10\beta+\gamma)s^3+10\gamma s^2-10K_r K_t s^2-10K_r(K+K_t Z_r)s-10K_r K Z_r \\ -10K_v K_t s^3-10K_v Z_v K_t s^2-10K_v K s^2-10K_v Z_v K s = 0$$

or

$$s^5+(10+\beta)s^4+[(10\beta+\gamma)-10K_v K_t]s^3 \\ +[-10K_v K-(10K_r+10K_v Z_v)K_t+10\gamma]s^2 \\ -[(-10K_v Z_v-10K_r)K-10K_r Z_r K_t]s+(-10K_r Z_r)K = 0 \quad (4.19)$$

Considering again the Class D surface ship and evaluating Eqs. (3.30), according to the values of Table II gives:

$$\begin{aligned} K_r &= -1.74425 \\ Z_r &= 1.24744 \\ K_v &= 0.32126 \\ Z_v &= 2.288 \\ \beta &= 3.227 \\ \gamma &= 2.0 \end{aligned}$$

And now Eq. (4.19) becomes:

$$s^5 + 32.27s^4 + [-3.2126K_t + 34.27]s^3 + [-3.2126K + 10.0921K_t + 20.0]s^2 + [10.0921K + 21.7584K_t]s + 21.784K = 0 \quad (4.20)$$

The PARAM "A" computer program was simulated for the case of Eq. (4.20) and for $U_0' = 1.0$ and a sample of the computer result containing permissible pairs of values of K and K_t is given in Table IV.

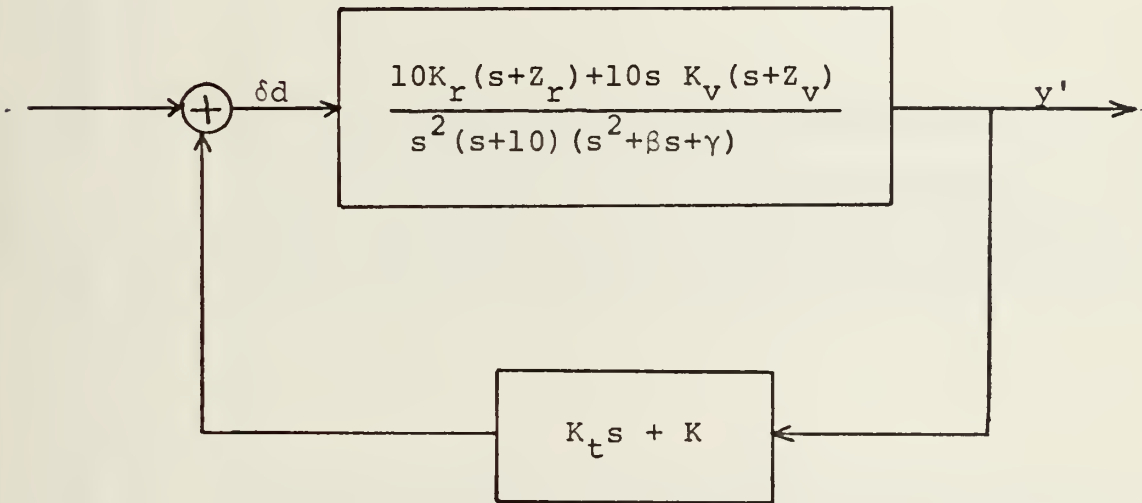


Figure 14. The Equivalent Distance Control Loop

4. Computer Simulation

Two different problems were encountered:

a. The station keeping case

The ship, due to external disturbances, has acquired a certain sway, $y = 0.1$ and a certain sway rate,

TABLE IV
APPROPRIATE VALUES OF PARAMETERS K AND K_t

K	K_t	Ω	ζ	Third Parameter
0.89184E-06	0.58816E-04	0.12927E-03	0.80000E-00	0.13665E-08
0.96373E-06	0.62283E-04	0.13244E-03	0.80000E-00	0.13665E-08
0.10413E-07	0.65935E-04	0.13568E-03	0.80000E-00	0.13665E-08
0.11249E-07	0.69782E-04	0.13901E-03	0.80000E-00	0.13665E-08
0.72336E-02	0.15389E-00	0.10000E-01	0.10000E-01	0.13665E-08
0.75513E-02	0.15703E-00	0.10245E-01	0.10000E-01	0.13665E-08
0.78822E-02	0.16022E-00	0.10496E-01	0.10000E-01	0.13665E-08
0.82269E-02	0.16346E-00	0.10753E-01	0.10000E-01	0.13665E-08
0.85857E-02	0.16676E-00	0.11017E-01	0.10000E-01	0.13665E-08
0.89593E-02	0.17011E-00	0.11286E-01	0.10000E-01	0.13665E-08
0.93483E-02	0.17351E-00	0.11563E-01	0.10000E-01	0.13665E-08
0.97533E-02	0.17697E-00	0.11846E-01	0.10000E-01	0.13665E-08
0.10175E-01	0.18049E-00	0.12137E-01	0.10000E-01	0.13665E-08
0.10614E-01	0.18406E-00	0.12434E-01	0.10000E-01	0.13665E-08
0.11070E-01	0.18769E-00	0.12738E-01	0.10000E-01	0.13665E-08
0.11546E-01	0.19138E-00	0.13051E-01	0.10000E-01	0.13665E-08
0.12041E-01	0.19512E-00	0.13370E-01	0.10000E-01	0.13665E-08
0.12556E-01	0.19893E-00	0.13698E-01	0.10000E-01	0.13665E-08
0.13092E-01	0.20280E-00	0.14033E-01	0.10000E-01	0.13665E-08
0.13650E-01	0.20673E-00	0.14377E-01	0.10000E-01	0.13665E-08
0.14232E-01	0.21072E-00	0.14730E-01	0.10000E-01	0.13665E-08
0.14837E-01	0.21478E-00	0.15090E-01	0.10000E-01	0.13665E-08
0.15467E-01	0.21890E-00	0.15460E-01	0.10000E-01	0.13665E-08
0.16123E-01	0.22310E-00	0.15839E-01	0.10000E-01	0.13665E-08
0.16807E-01	0.22736E-00	0.16227E-01	0.10000E-01	0.13665E-08
0.17519E-01	0.23170E-00	0.16624E-01	0.10000E-01	0.13665E-08
0.18262E-01	0.23611E-00	0.17032E-01	0.10000E-01	0.13665E-08
0.19036E-01	0.24060E-00	0.17449E-01	0.10000E-01	0.13665E-08
0.19844E-01	0.24518E-00	0.17877E-01	0.10000E-01	0.13665E-08
0.20686E-01	0.24983E-00	0.18315E-01	0.10000E-01	0.13665E-08
0.21566E-01	0.25458E-00	0.18763E-01	0.10000E-01	0.13665E-08
0.22485E-01	0.25942E-00	0.19223E-01	0.10000E-01	0.13665E-08
0.23446E-01	0.26436E-00	0.19694E-01	0.10000E-01	0.13665E-08
0.24451E-01	0.26940E-00	0.20176E-01	0.10000E-01	0.13665E-08
0.25503E-01	0.27455E-00	0.20671E-01	0.10000E-01	0.13665E-08
0.26606E-01	0.27982E-00	0.21177E-01	0.10000E-01	0.13665E-08
0.27762E-01	0.28521E-00	0.21696E-01	0.10000E-01	0.13665E-08
0.28976E-01	0.29074E-00	0.22227E-01	0.10000E-01	0.13665E-08
0.30252E-01	0.29641E-00	0.22772E-01	0.10000E-01	0.13665E-08
0.31594E-01	0.30223E-00	0.23330E-01	0.10000E-01	0.13665E-08

$\dot{y} = 0.01$ (it has been moved from its original position laterally by: $y' \times L$ feet, and the disturbance is continuing).

The computer program III was simulated for the Class D ship, for $U_0' = 1.0$, and for an arbitrary pair of $K = 0.0225$ and $K_t = 0.25942$ of Table III, corresponding to a critically damped ($\zeta=1.0$) response. The result is shown in Fig. 15.

b. The Station Changing Case

The ship is originally on a straightforward motion at a specified course. It is desired to move to a course parallel to the original one, but at a distance $D = D' \times L$ from its original position laterally, as shown in Fig. 16.

The control loop of Fig. 13 is now modified as shown in Fig. 17 by the addition of the input reference $-D'$. The input reference is given a negative sign because for a desired lateral translation of the ship to its starboard direction, a negative rudder δ is required. It is now clear that the ordered rudder angle becomes $\delta_d = K(y' - D') + K_t \cdot \dot{y}'$, and therefore the computer program III is modified accordingly to yield computer program IV.

For this station changing case, computer program IV was simulated, for $U_0' = 1.0$ of the Class D ship and for the same pair of $K = 0.0225$ and $K_t = 0.25942$ of Table III and the result is shown in Fig. 18.

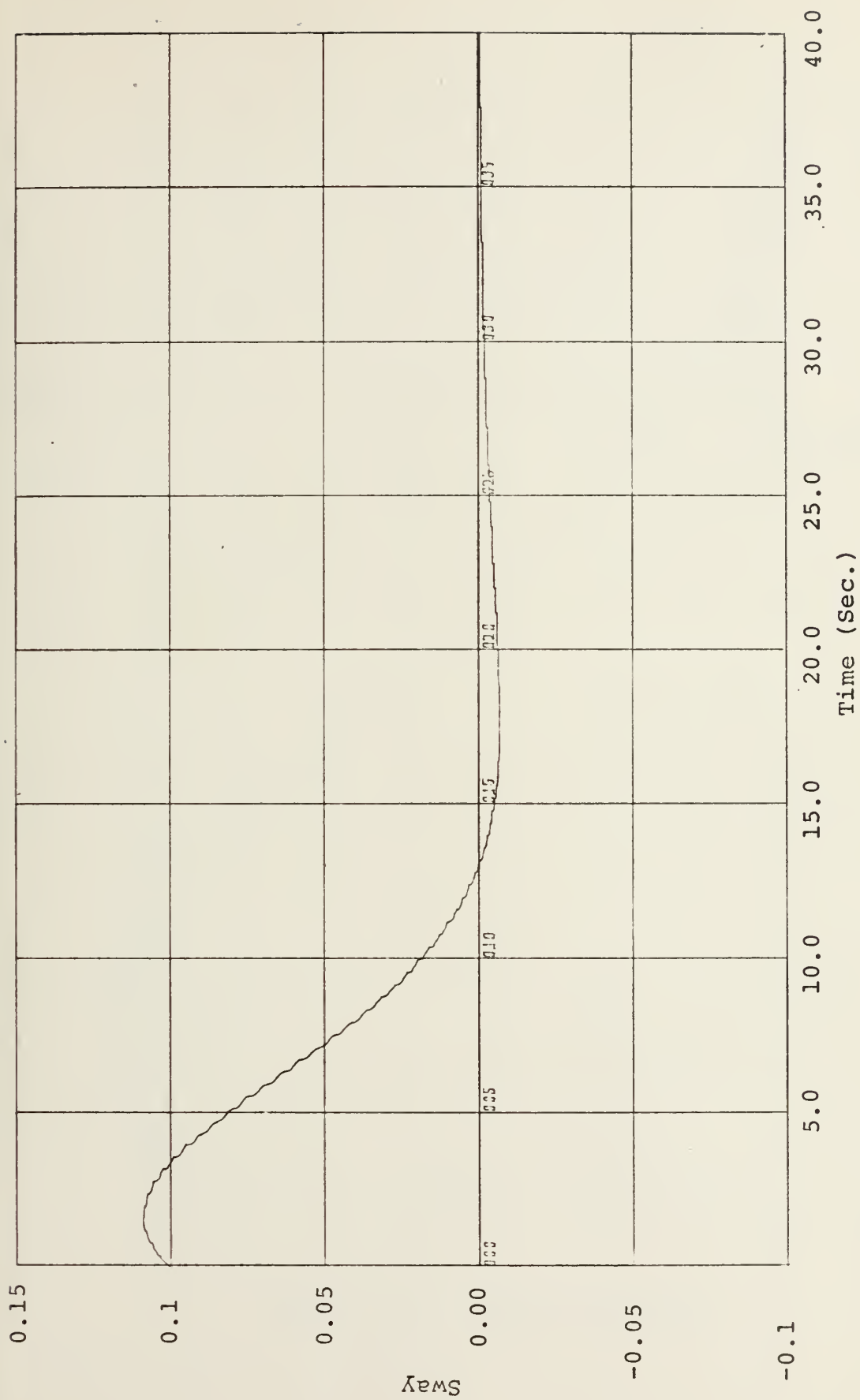


Figure 15. The Station Keeping System's Response

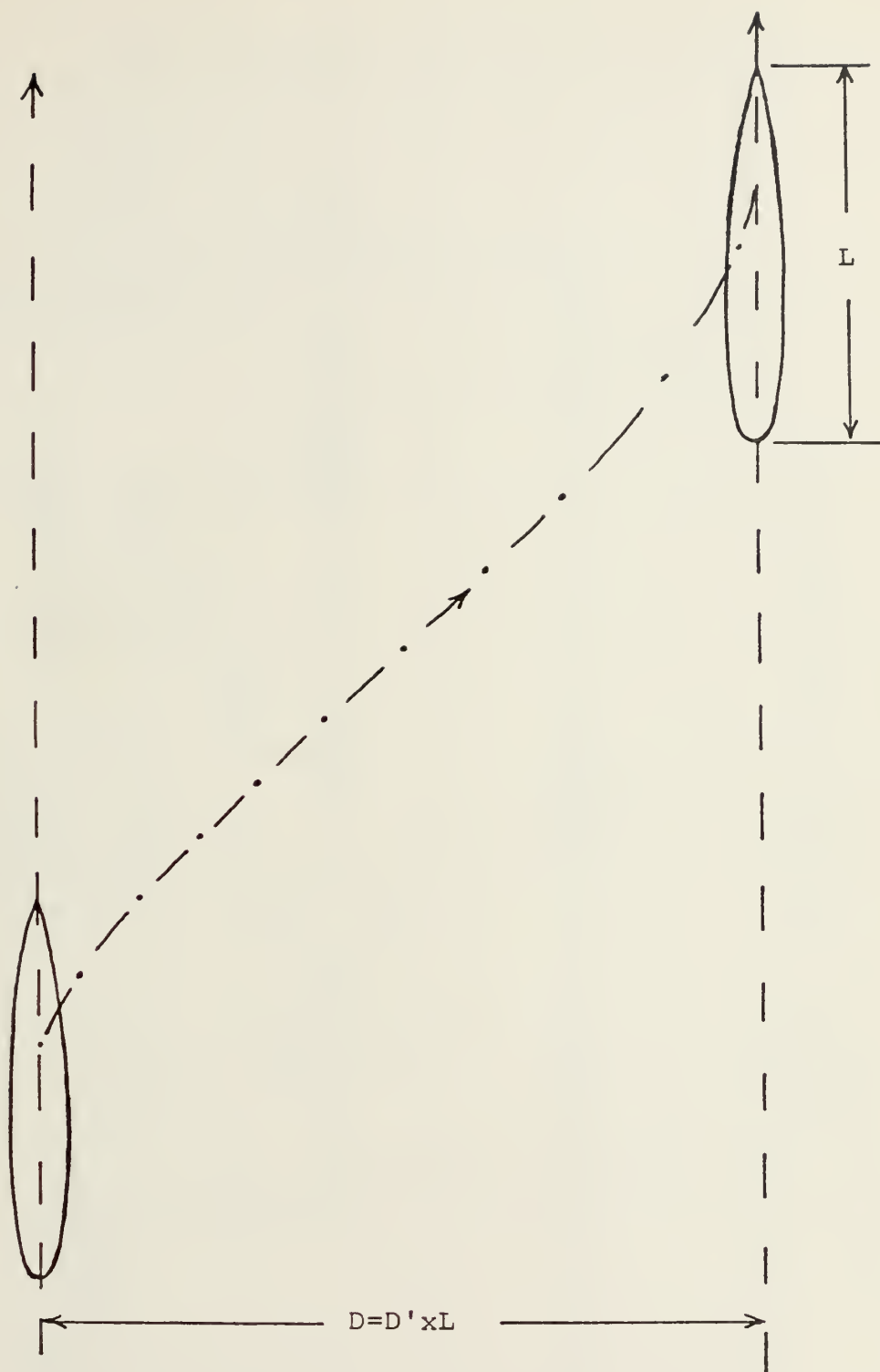


Figure 16. Station Changing

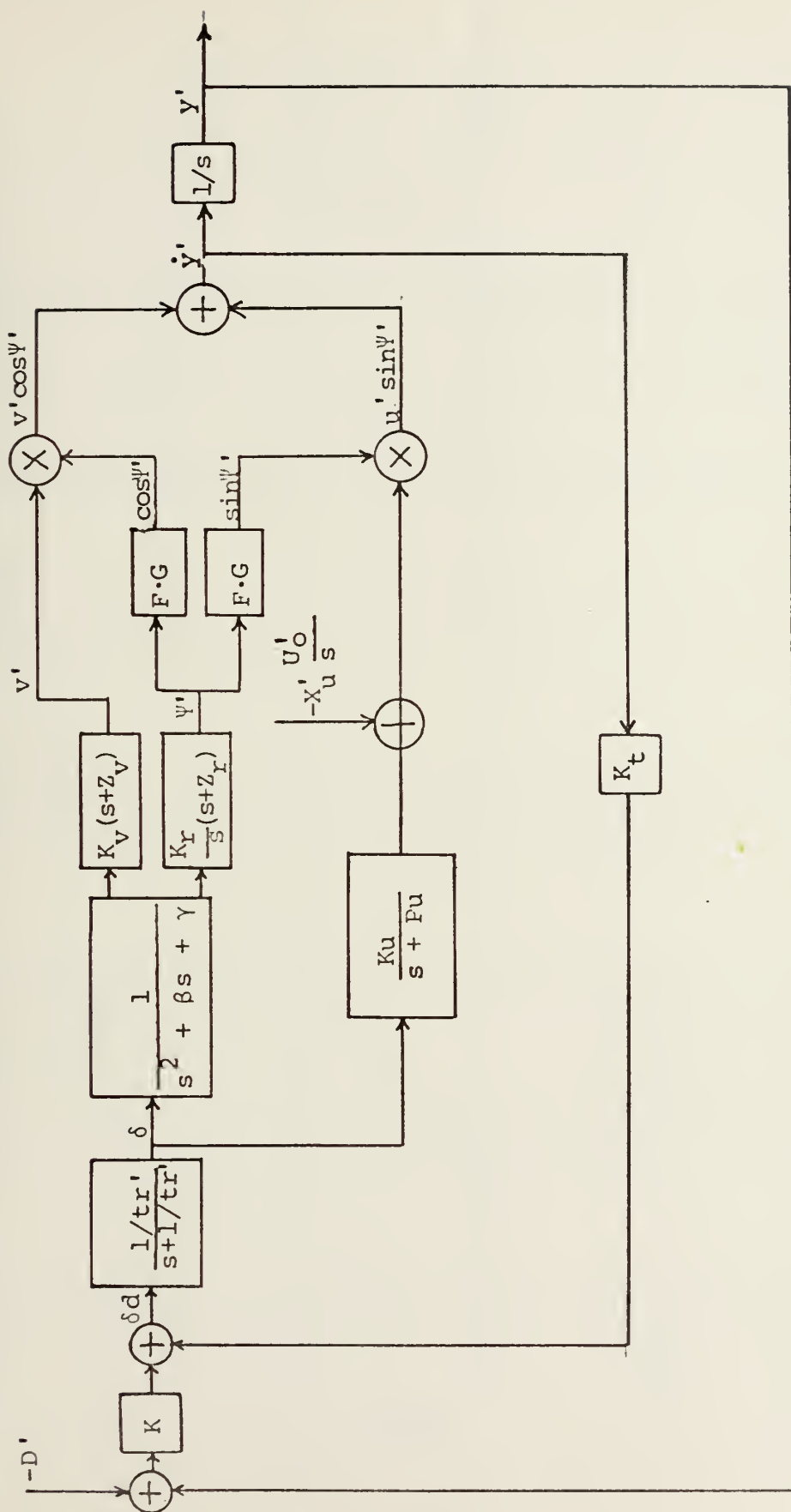


Figure 17. The Distance Changing Control Loop.

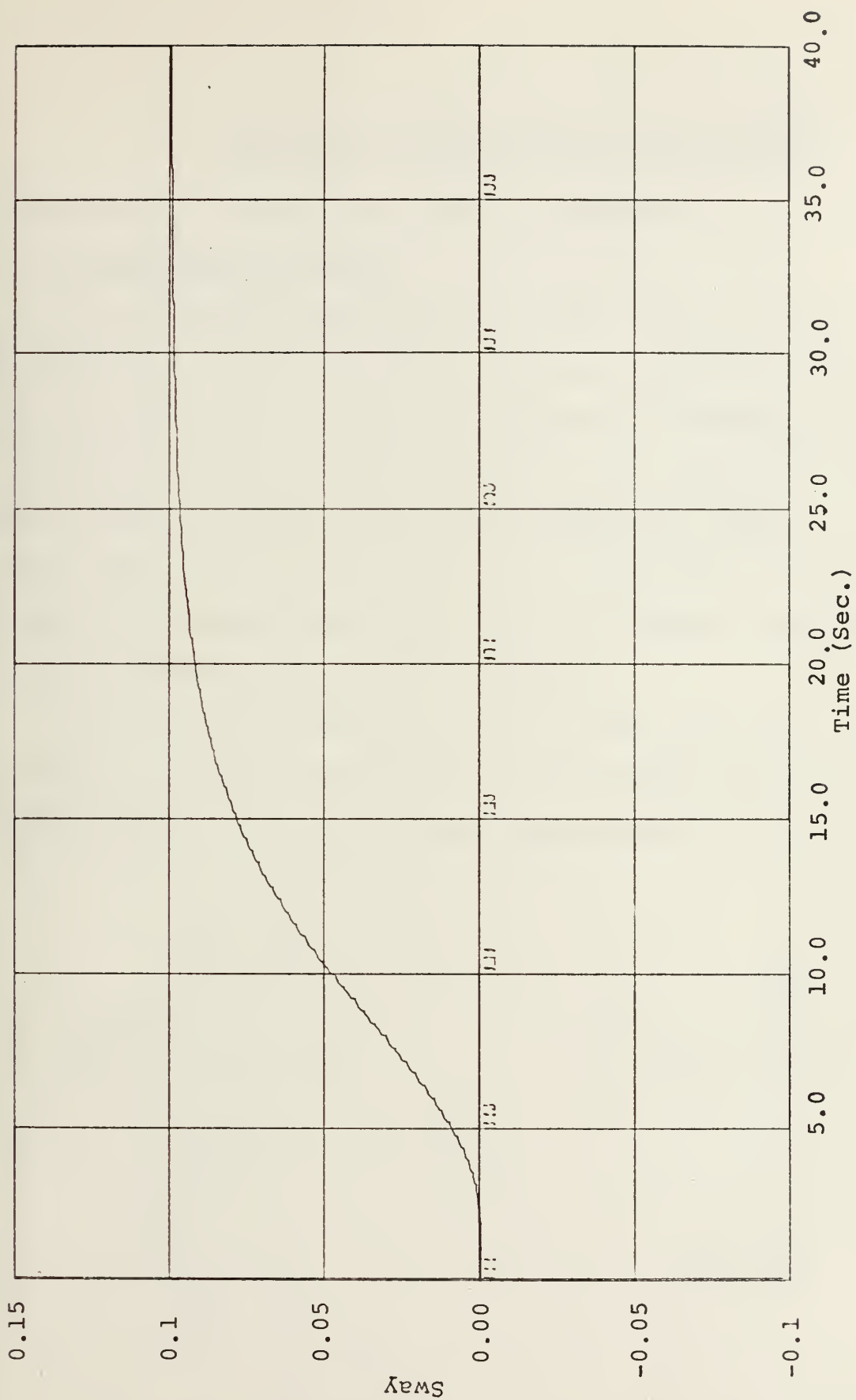


Figure 18. The Station-Changing System's Response

V. AN ACTUAL REPLENISHMENT AT SEA PROBLEM

A. INCLUSION OF INTERACTION FORCES AND MOMENTS

1. Characteristic Values

During Newton's experiment [3], for each position of one ship relative to the other, two forces were measured on each F_1 and F_2 as in Fig. 19. These two forces can be represented by a single force $F_I = F_1 + F_2$ and a moment M_I , the magnitude of which depends upon the position about which moments are taken. If any of those forces are to be included in the equations of motion, they have to be represented by a force and a moment about the origin, and therefore the center of gravity.

All forces and moments in Figs. 1 and 3 are with respect to a point, called the "neutral point N" at which neglecting transient effects, any lateral force applied will cause no change

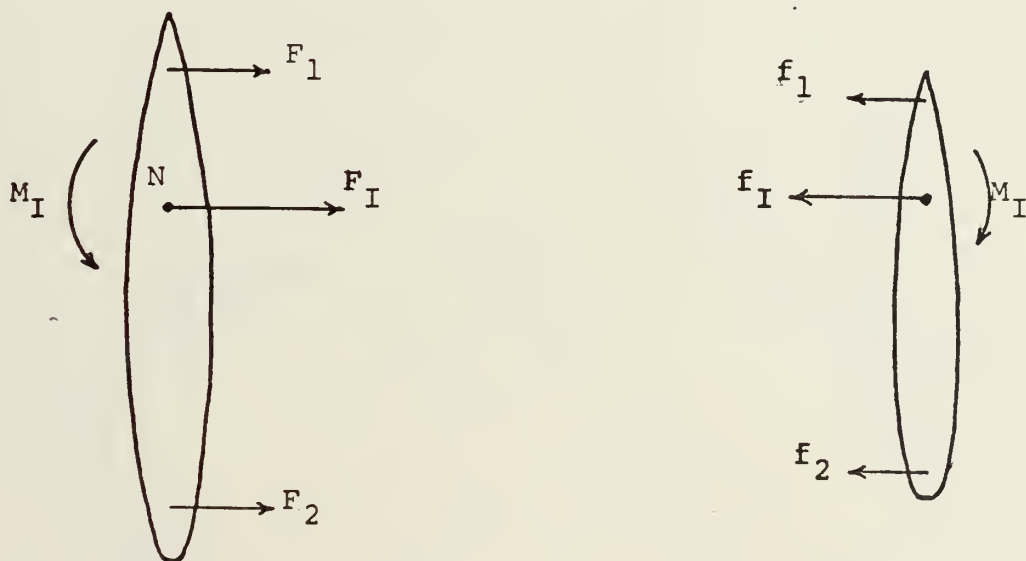


Figure 19. The Interactive Forces and Moments

in heading, although it will cause a change in course. It has experimentally been found that the neutral point lies at 1/5 the length of the ship from the bow.

From Fig. 3 (100 ft. lateral separation) the non-dimensional force and moment of one of the two ships at the exactly abeam position are respectively

$$F_{I(N)} = 2.16 \cdot 10^{-4} \text{ and } M_{I(N)} = -0.8 \cdot 10^{-4} .$$

If the center of gravity is assumed to be at the geometric center of symmetry of the ship, then the forces F_1 and F_2 equivalent to $F_{I(N)}$ and $M_{I(N)}$ can be represented by a single force $F_{I(G)}$ and a moment $M_{I(G)}$ about the center of gravity that can be found to be:

$$F_{I(G)} = 2.16 \cdot 10^{-4} \text{ and } M_{I(G)} = -0.152 \cdot 10^{-4}$$

as shown in Fig. 20.

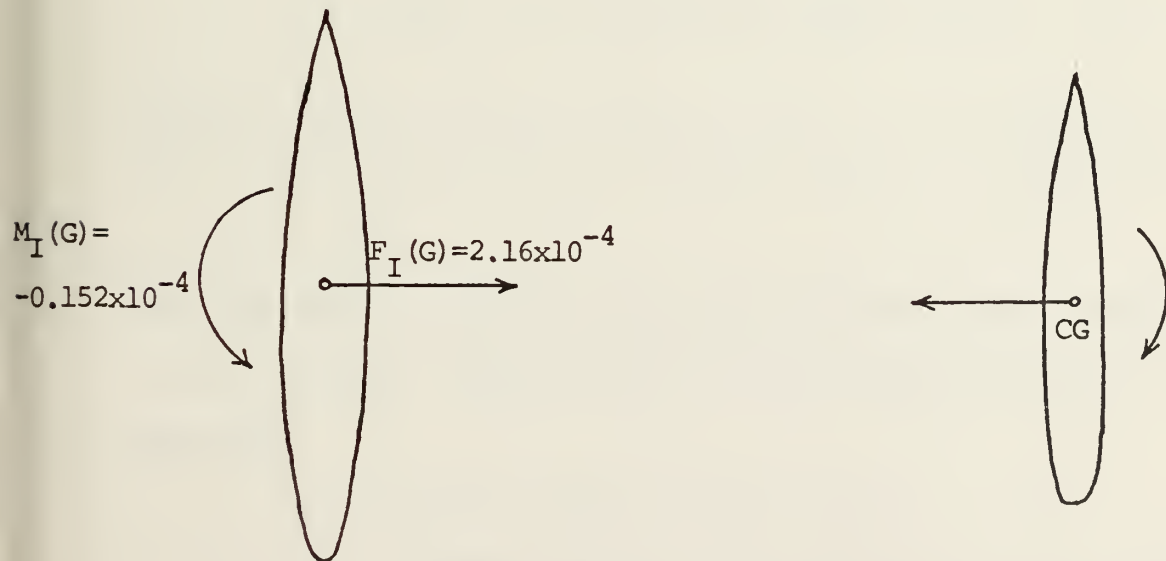


Figure 20. Equivalent Forces and Moments.

2. Modification of the Equations of Motion

If YI and NI are the nondimensionalized force and moment acting on a ship when exactly alongside another, then the equations for this ship become (for $U'_0 = 1.0$):

$$\frac{v'(s)}{s} [\alpha_{aA} s^2 + \beta_{aA} s + \gamma_{aA}] + \psi'(s) [\alpha_{bA} s^2 + \beta_{bA} s + \gamma_{bA}] = Y'_\delta \cdot \delta(s) + YI(s)$$

$$\frac{v'(s)}{s} [\alpha_{aB} s^2 + \beta_{aB} s + \gamma_{aB}] + \psi'(s) [\alpha_{bB} s^2 + \beta_{bB} s + \gamma_{bB}] = N'_\delta \cdot \delta(s) + NI(s)$$

$$\frac{u'(s)}{s} [\alpha_{cC} s^2 + \beta_{cC} s + \gamma_{cC}] = X'_\delta \cdot \delta(s) - Xu'$$

Since $\delta(s) = 10\delta d(s)/s+10$, then Eqs. (5.1) can be written:

$$\begin{aligned} \frac{v'(s)}{s} [\alpha_{aA} s^3 + (\beta_{aA} + 10\alpha_{aA}) s^2 + 10\beta_{aA} s + 10\gamma_{aA}] \\ + \psi'(s) [\alpha_{bA} s^3 + (\beta_{bA} + 10\alpha_{bA}) s^2 + 10\beta_{bA} s + 10\gamma_{bA}] = 10Y'_\delta \cdot \delta d(s) + YI(s) \end{aligned}$$

$$\begin{aligned} \frac{v'(s)}{s} [\alpha_{aB} s^3 + (\beta_{aB} + 10\alpha_{aB}) s^2 + 10\beta_{aB} s + 10\gamma_{aB}] \\ + \psi'(s) [\alpha_{bB} s^3 + (\beta_{bB} + 10\alpha_{bB}) s^2 + 10\beta_{bB} s + 10\gamma_{bB}] = 10N'_\delta \cdot \delta d(s) + NI(s) \end{aligned}$$

$$\begin{aligned} \frac{u'(s)}{s} [\alpha_{cC} s^3 + (\beta_{cC} + 10\alpha_{cC}) s^2 + 10\beta_{cC} s + 10\gamma_{cC}] = 10X'_\delta \cdot \delta d(s) - 10X'_u \end{aligned} \quad (5.2)$$

The following important assumptions will become the basis of all further investigation of the replenishment at sea problem:

- a. The two ships are identical.
- b. All hydrodynamic coefficients are not affected by the intermingling of the water pressures between the ships, and the motions of the ships, and therefore they remain constant.

- c. The two ships are considered already as being alongside each other.
- d. The forces and moments acting on the ships are identical (in absolute value), and they are as shown in Fig. 20.
- e. The changes of YI and NI , due to relative motions of the ships, are considered negligible and $\dot{YI} = \dot{NI} = 0$.
- f. The subscript 1 attached to the quantities of the equations of motion characterizes the replenishing ship, and correspondingly, subscript 2 indicates quantities referring to the receiving ship.
- g. A pair of values of the parameters K and K_t of either Table III or IV will be given the definition "optimal," in the sense that it constitutes a proper pair of parameter values, for which the performance of the system corresponds to a $\zeta=1.0$ condition.

Therefore under the above assumptions the equations for replenishing ship should be:

$$\begin{aligned}
 \frac{v_1'(s)}{s} [\alpha_{aA}' s^3 + \beta_{aA}' s^2 + \gamma_{aA}' s + \delta_{aA}'] + \psi_1'(s) [\alpha_{bA}' s^3 + \beta_{bA}' s^2 + \gamma_{bA}' s + \delta_{bA}'] \\
 = 10Y_{\delta}' \cdot \delta d_1(s) + YI1 \\
 \frac{v_1'(s)}{s} [\alpha_{aB}' s^3 + \beta_{aB}' s^2 + \gamma_{aB}' s + \delta_{aB}'] + \psi_1'(s) [\alpha_{bB}' s^3 + \beta_{bB}' s^2 + \gamma_{bB}' s + \delta_{bB}'] \quad (5.3) \\
 = 10N_{\delta}' \cdot \delta d_1(s) + NI1 \\
 \frac{u_1'(s)}{s} [\alpha_{cC}' s^3 + \beta_{cC}' s^2 + \gamma_{cC}' s + \delta_{cC}'] = 10X_{\delta}' \cdot \delta d_1(s) - 10Xu'
 \end{aligned}$$

Or setting

$$\frac{v_1'(s)}{s} = A_1(s) \quad \text{or} \quad v_1'(t) = \dot{A}_1$$

$$\psi_1'(s) = B_1(s) \quad \text{or} \quad \psi_1'(t) = B_1$$

$$\frac{u_1'(s)}{s} = C_1(s) \quad \text{or} \quad u_1'(t) = \dot{C}_1$$

gives

$$\begin{aligned} \alpha_{aA}' \ddot{\ddot{A}}_1 + \beta_{aA}' \ddot{\ddot{A}}_1 + \gamma_{aA}' \dot{\ddot{A}}_1 + \alpha_{bA}' \ddot{\ddot{B}}_1 + \beta_{bA}' \ddot{\ddot{B}}_1 + \gamma_{bA}' \dot{\ddot{B}}_1 &= IF1_1 \\ \alpha_{aB}' \ddot{\ddot{A}}_1 + \beta_{aB}' \ddot{\ddot{A}}_1 + \gamma_{aB}' \dot{\ddot{A}}_1 + \alpha_{bB}' \ddot{\ddot{B}}_1 + \beta_{bB}' \ddot{\ddot{B}}_1 + \gamma_{bB}' \dot{\ddot{B}}_1 &= IF2_1 \\ \alpha_{cC}' \ddot{\ddot{C}}_1 + \beta_{cC}' \ddot{\ddot{C}}_1 + \gamma_{cC}' \dot{\ddot{C}}_1 &= IF3_1 \end{aligned} \quad (5.4)$$

where

$$\begin{aligned} IF1_1 &= 10Y_{\delta}' \cdot \delta d_1 + YI1 \\ IF2_1 &= 10N_{\delta}' \cdot \delta d_1 + NI1 \\ IF3_1 &= 10X_{\delta}' \cdot \delta d_1 - 10Xu' \end{aligned}$$

and for the primed coefficients the relations (4.14a) are valid.

Furthermore Eqs. (5.4) can be written:

$$\begin{aligned} \alpha_{aA}' \ddot{\ddot{A}}_1 + \alpha_{bA}' \ddot{\ddot{B}}_1 &= I1_1 \\ \alpha_{aB}' \ddot{\ddot{A}}_1 + \alpha_{bB}' \ddot{\ddot{B}}_1 &= I2_1 \\ \alpha_{cC}' \ddot{\ddot{C}}_1 &= I3_1 \end{aligned} \quad (5.5)$$

where

$$I1_1 = IF1_1 - \beta'_{aA} \cdot \ddot{A}_1 - \gamma'_{aA} \dot{A}_1 - \beta'_{bA} \ddot{B}_1 - \gamma'_{bA} \dot{B}_1$$

$$I2_1 = IF2_1 - \beta'_{aB} \cdot \ddot{A}_1 - \gamma'_{aB} \dot{A}_1 - \beta'_{bB} \ddot{B}_1 - \gamma'_{bB} \dot{B}_1$$

$$I3_1 = IF3_1 - \beta'_{cC} \ddot{C}_1 - \gamma'_{cC} \dot{C}_1$$

In the same fashion equations for the receiving ship can be found to be, for $U'_0 = 10$:

$$\alpha'_{aA} \ddot{A}_2 + \beta'_{aA} \ddot{A}_2 + \gamma'_{aA} \dot{A}_2 + \alpha'_{bA} \ddot{B}_2 + \beta'_{bA} \ddot{B}_2 + \gamma'_{bB} \dot{B}_2 = IF1_2$$

$$\alpha'_{aB} \ddot{A}_2 + \beta'_{aB} \ddot{A}_2 + \gamma'_{aA} \dot{A}_2 + \alpha'_{bB} \ddot{B}_2 + \beta'_{bB} \ddot{B}_2 + \gamma'_{bB} \dot{B}_2 = IF2_2 \quad (5.6)$$

$$\alpha'_{cC} \ddot{C}_2 + \beta'_{cC} \ddot{C}_2 + \gamma'_{cC} \dot{C}_2 = IF3_2$$

where

$$IF1_2 = 10Y'_0 \cdot \delta d + YI2$$

$$IF2_2 = 10N'_0 \cdot \delta d + NI2$$

$$IF3_2 = 10X'_0 \cdot \delta d - 10Xu'$$

or

$$\alpha'_{aA} \ddot{A}_2 + \alpha'_{bA} \ddot{B}_2 = I1_2$$

$$\alpha'_{aB} \ddot{A}_2 + \alpha'_{bB} \ddot{B}_2 = I2_2 \quad (5.7)$$

$$\alpha'_{cC} \ddot{C}_2 = I3_2$$

where

$$I1_2 = IF1_2 - \beta'_{aA} \ddot{A}_2 - \gamma'_{aA} \dot{A}_2 - \beta'_{bA} \ddot{B}_2 - \gamma'_{bA} \dot{B}_2$$

$$I2_2 = IF2_2 - \beta'_{aB} \ddot{A}_2 - \gamma'_{aB} \dot{A}_2 - \beta'_{bB} \ddot{B}_2 - \gamma'_{bB} \dot{B}_2$$

$$I3_2 = IF3_2 - \beta'_{cC} \ddot{C}_2 - \gamma'_{cC} \dot{C}_2$$

3. The Behavior of the Ships Under the Influence of Interactive Forces and Moments Without Controls.

Equations (5.7) give computer program V. This computer program in which $\delta d_2 = DD2 = 0$, was simulated for the class D ship, for $U'_{0_2} = 1.0$ and the results showing the sway and yaw of the receiving ship are shown in Figs. 21 and 22, respectively. If the replenishing ship were simulated instead, with $\delta d_1 = DD1 = 0$, the results would have been identical but with a sign reversed.

B. THE PROPOSED METHODS OF CONTROL

1. Method I

In this method of controlling the maneuvering of the two ships involved in the replenishment at sea operation, the control of the replenishing ship should be made sensitive to the measured quantities of its yaw and yaw rate, and the control of the receiving ship should be made sensitive to the measured quantities of its yaw, yaw rate, relative sway and relative sway rate. If this is the case, the feedback control loop for the replenishing ship should be a course keeping one as shown in Fig. 9. The relative sway and relative sway rate could be made available by means of one or two sensing devices

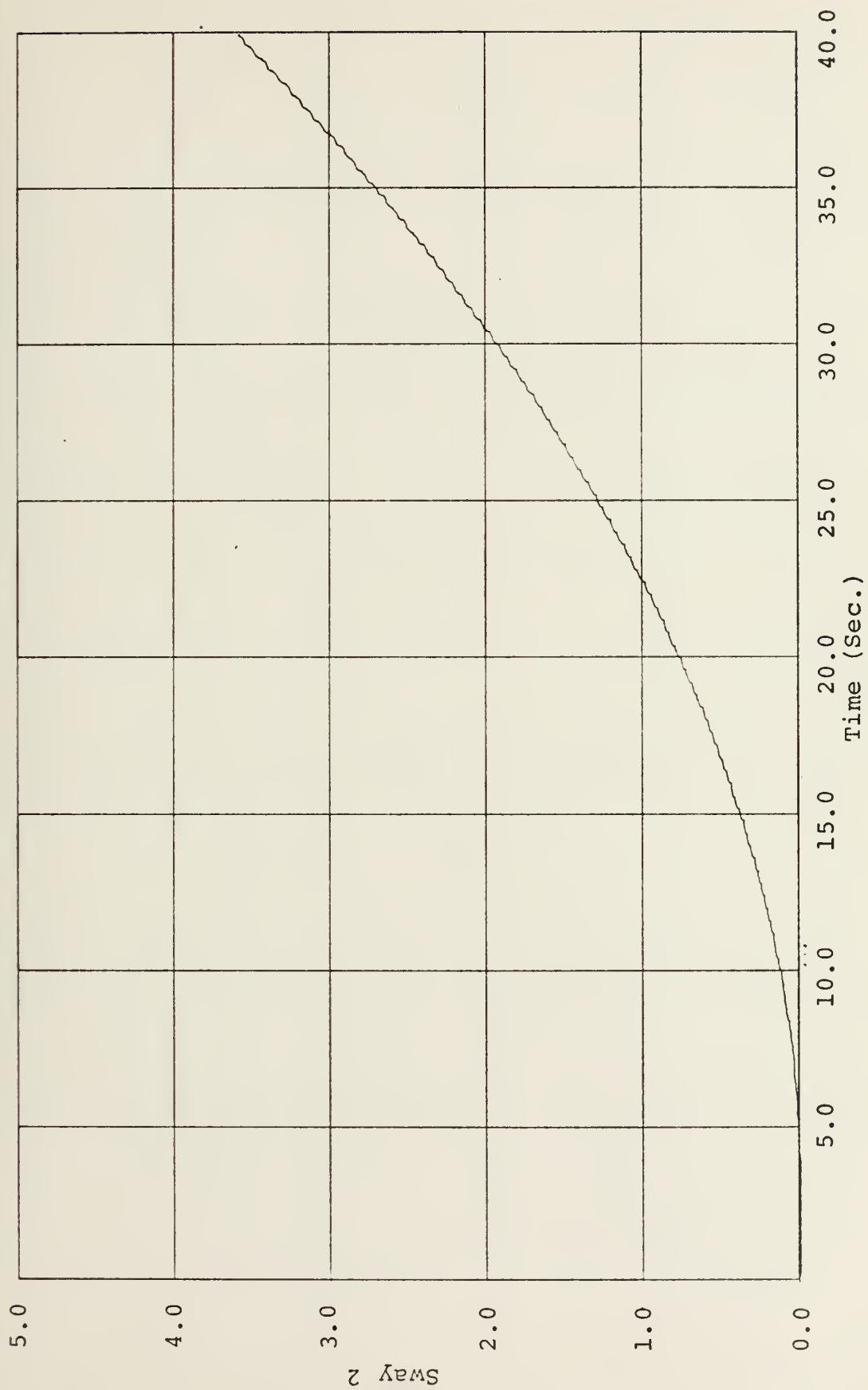


Figure 21. Sway 2 Vs. Time

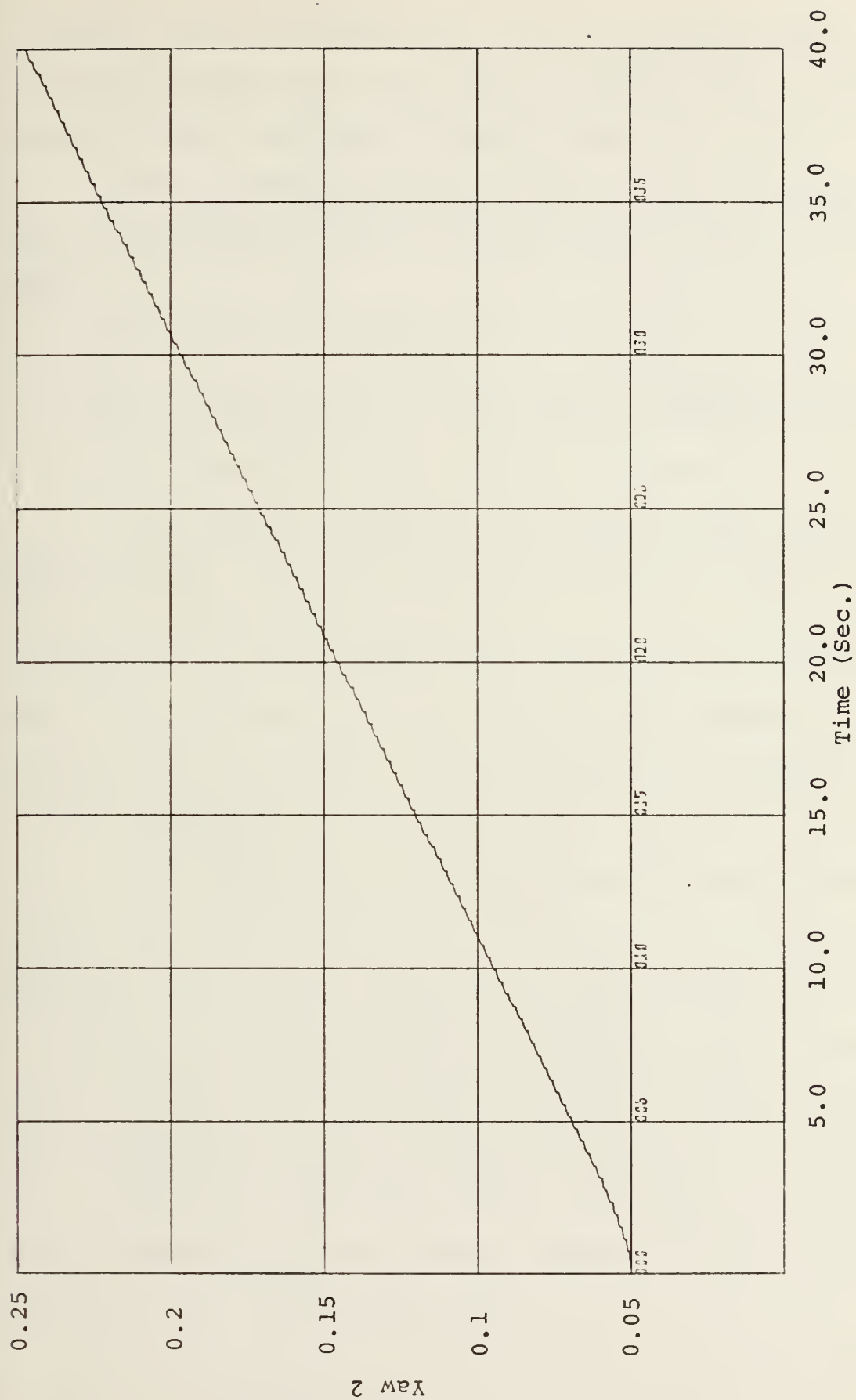


Figure 22. Yaw 2 Vs. Time

(radars) aboard the receiving ship that would provide a continuing signaling of the actual distance and its rate of change between the ships as shown in Fig. 23.

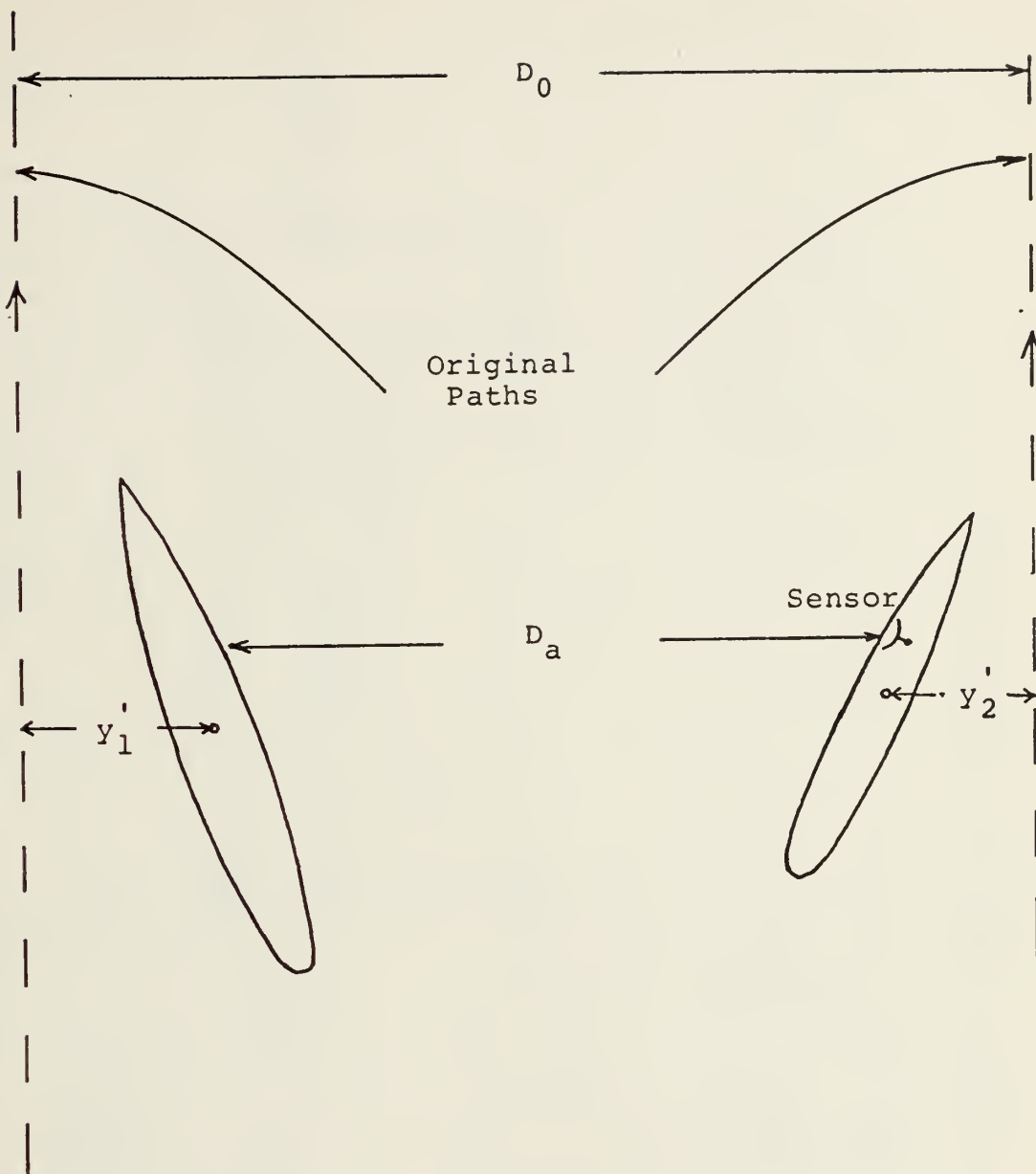
The feedback control loop for the receiving ship should be the one shown in Fig. 24. From Figs. 9 and 24 it is evident that

$$\begin{aligned}\delta d_1 &= DDC1 = (K_1 + K_{t_1}s)\Psi_1' = K1 \cdot B_1 + KT1 \cdot BD1 \\ \delta d_2 &= DDC2 + DDD2 = (K_2 + K_{t_2}s)\Psi_2' + (K_2' + K_{t_2}'s)(y_2' - y_1') \\ &= K2 \cdot B2 + KT2 \cdot BD2 + KP2 \cdot D + KTP2 \cdot DDOT\end{aligned}$$

The computer program VI was simulated for two class D ships with $U_{0_1}' = U_{0_2}' = 1.0$ and the results are shown in Figs. 25 through 31. It is noted here that K_1 and K_{t_1} are given the values corresponding to an optimal pair, whereas the pairs $K_2 - K_{t_2}$ and $K_2' - K_{t_2}'$ do not necessarily have to be optimal.

2. Method II

This method is exactly the same with the Method I except that now the values of K_1 and K_{t_1} for the replenishing ship are made slightly different than the optimal values used in Method I. In this case the replenishing ship is brought by the control action to such an equilibrium angle of yaw (Ψ_1'), that causes its heading to become not parallel to its original one. Then it is shown that the receiving ship is forced to keep proper separation between ships, keeping parallel to the new heading of the replenishing ship.



D_0 - Original desired separation

D_a - Actual distance measured by sensor

Relative sway $D = y_2' - y_1' = D_a - D_0$

Relative sway rate - \dot{D}_a

Figure 23. Relative Sway and Sway Rate.

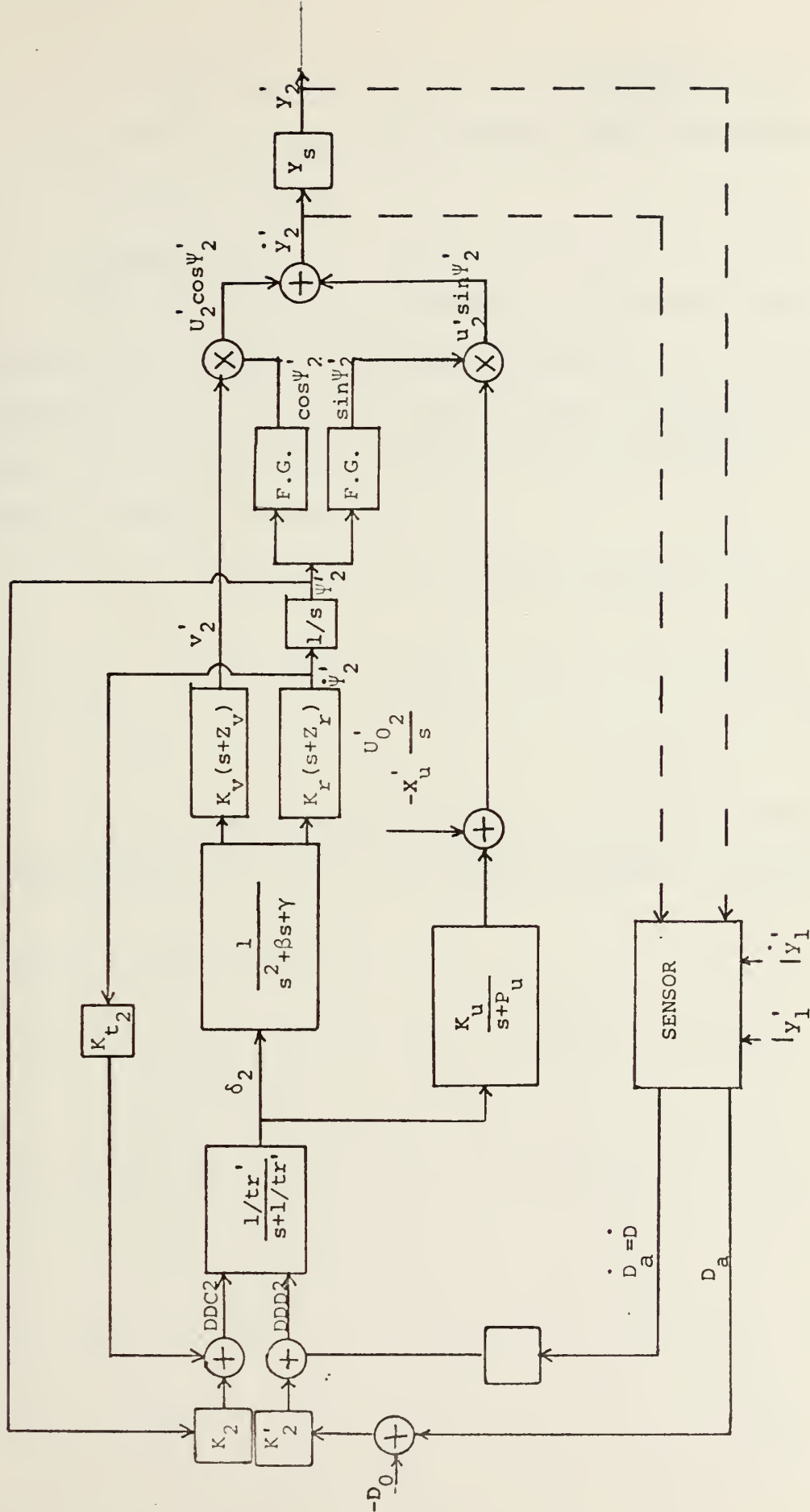


Figure 24. The Complete Course and Station Keeping Loop

The computer program VII was simulated for two class D ships, with $U'_{01} = U'_{02} = 1.0$ and the results are shown in Fig. 32 through Fig. 36.

3. Method III

In this method the controls of both the replenishing ship and the receiving ship are made sensitive to their respective yaw, yaw rate, relative sway and relative sway rate. In this case Fig. 24 applies for both ships and the feedback controls should be:

$$\begin{aligned}\delta_{d_1} &= DDC1 + DDD1 = (K_1 + K_{t_1}s)\psi'_1 + (K'_1 + K_{t_1}s)(y'_1 - y'_2) \\ &= K1 \cdot B1 + KT1 \cdot BD1 - KP1 \cdot D - KTP1 \cdot DDOT \\ \delta_{d_2} &= K2 \cdot B2 + KT2 \cdot BD2 + KP2 \cdot D + KTP2 \cdot DDOT\end{aligned}$$

The computer program VIII was simulated for two class D ships, with $U'_{01} = U'_{02} = 1.0$ and the results are shown in Fig. 37 through Fig. 41.

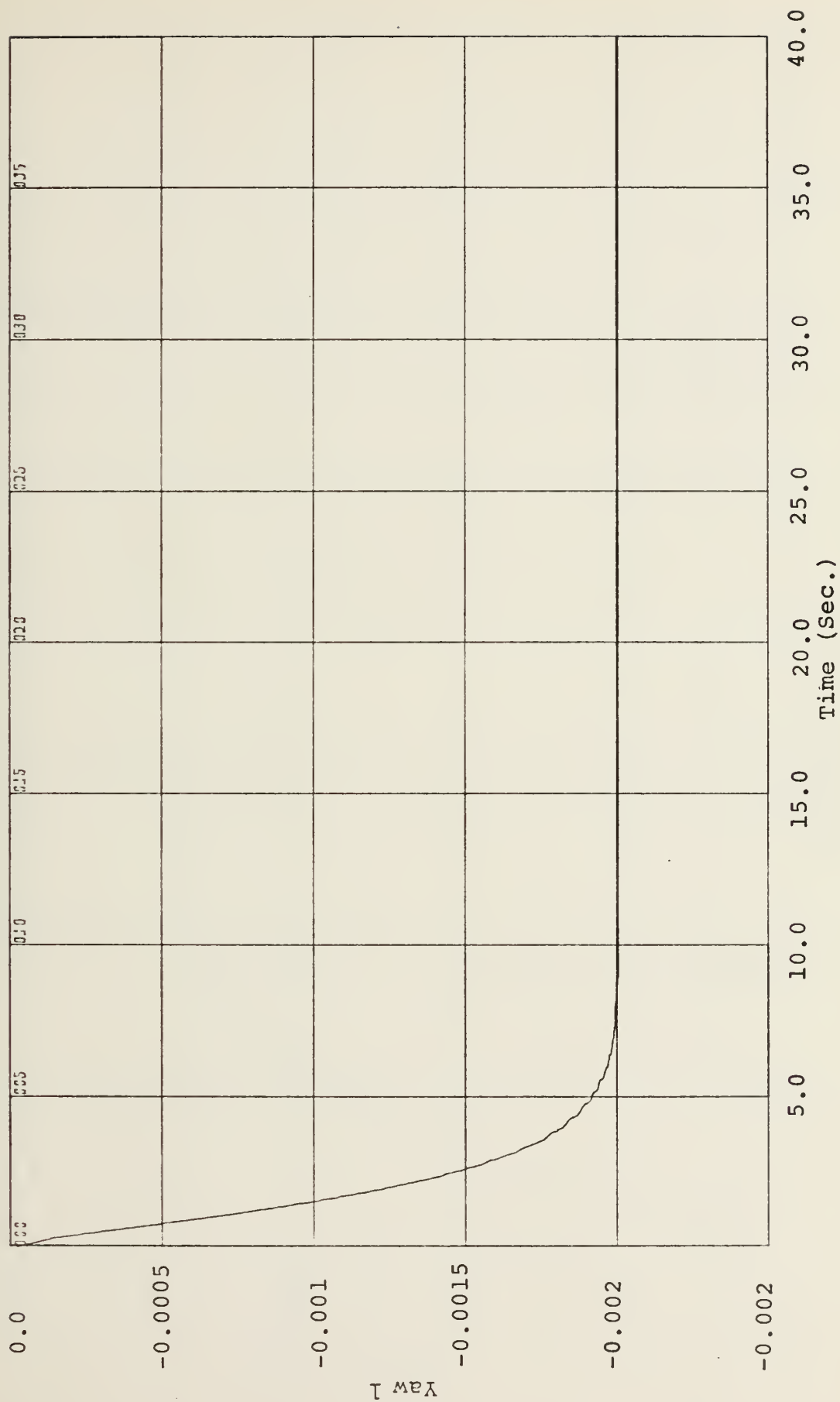


Figure 25. Yaw 1 Vs. Time.

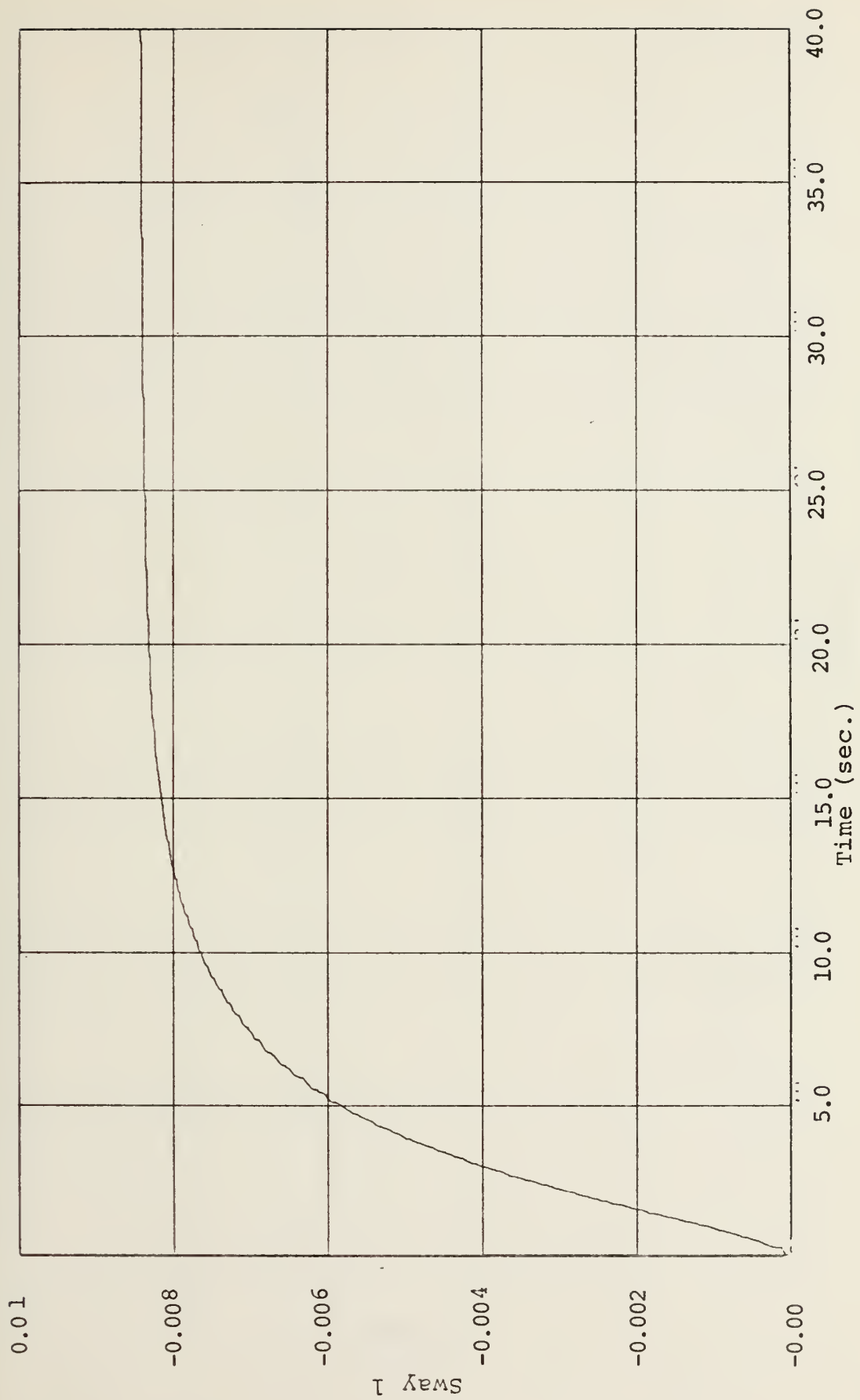


Figure 26. Sway 1 Vs. Time

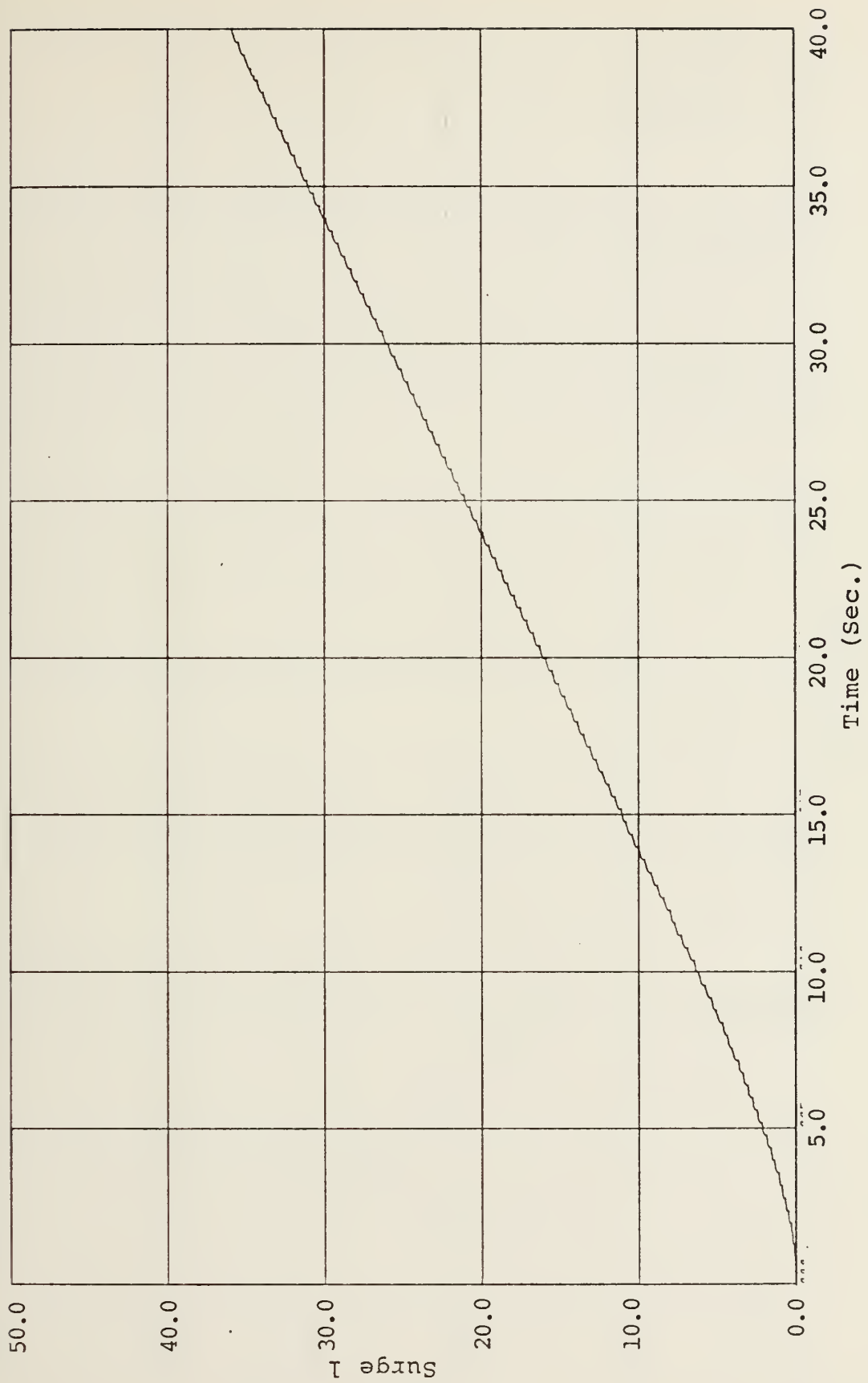


Figure 27. Surge 1 Vs. Time

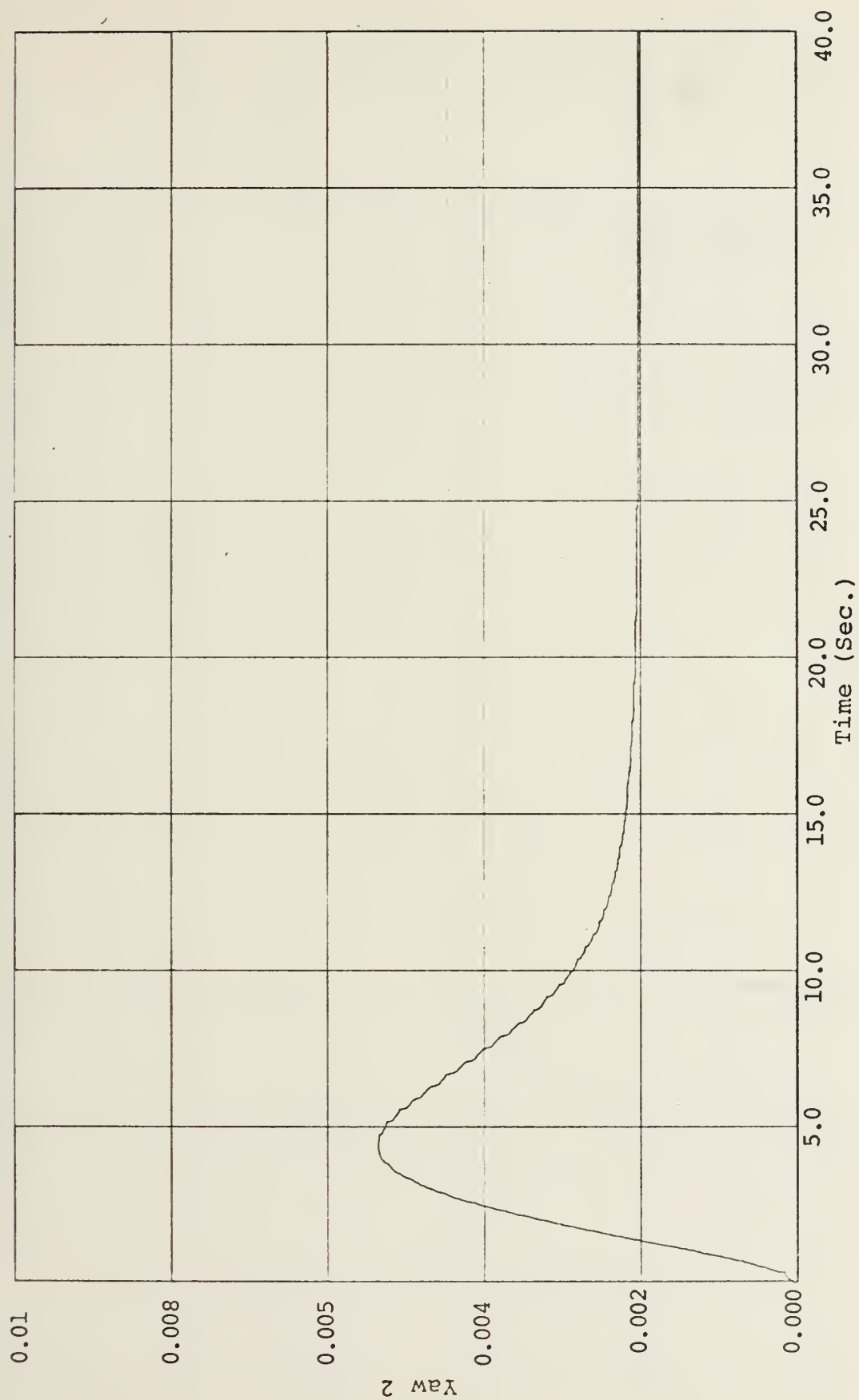


Figure 28. Yaw 2 Vs. Time

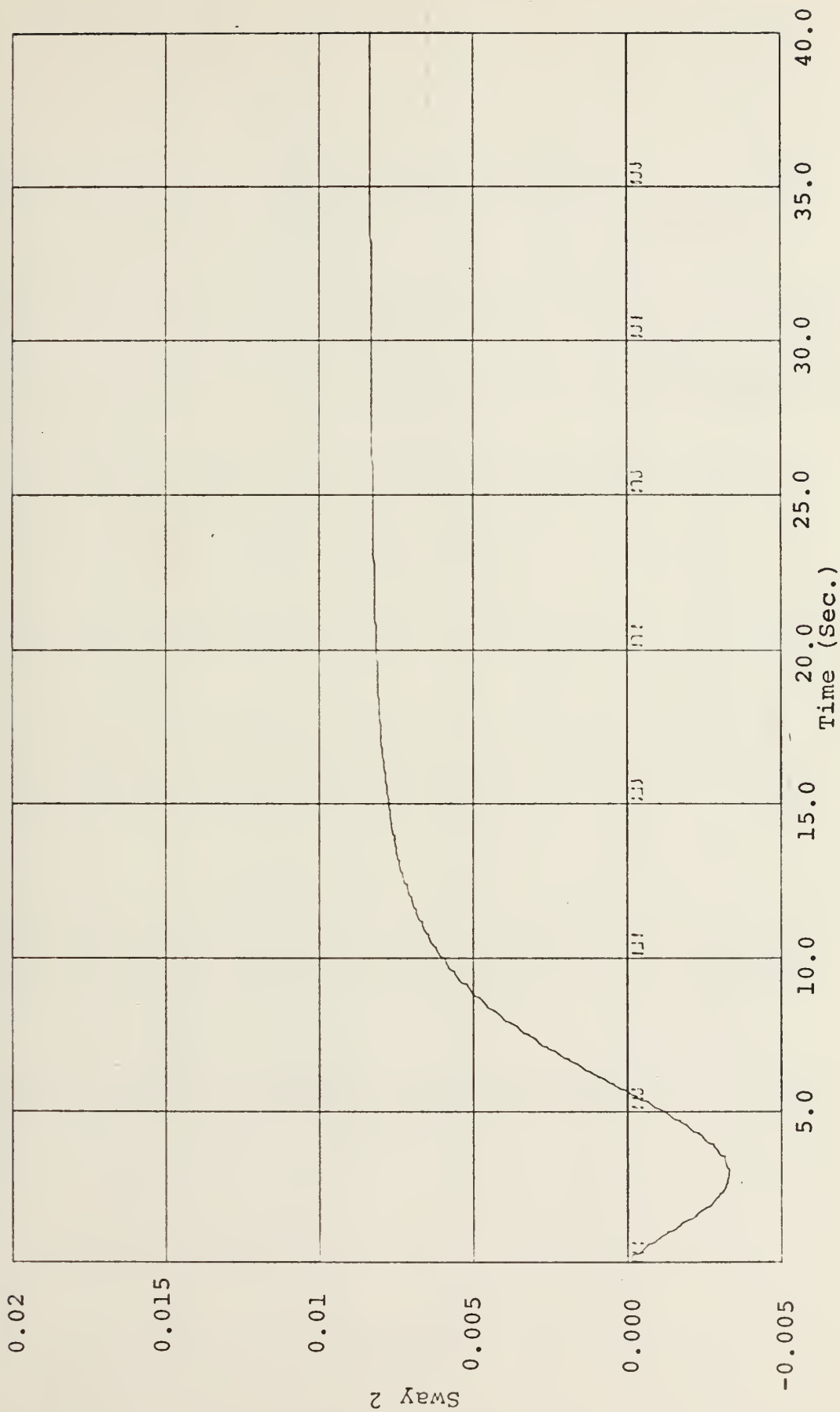


Figure 29. Sway 2 Vs. Time

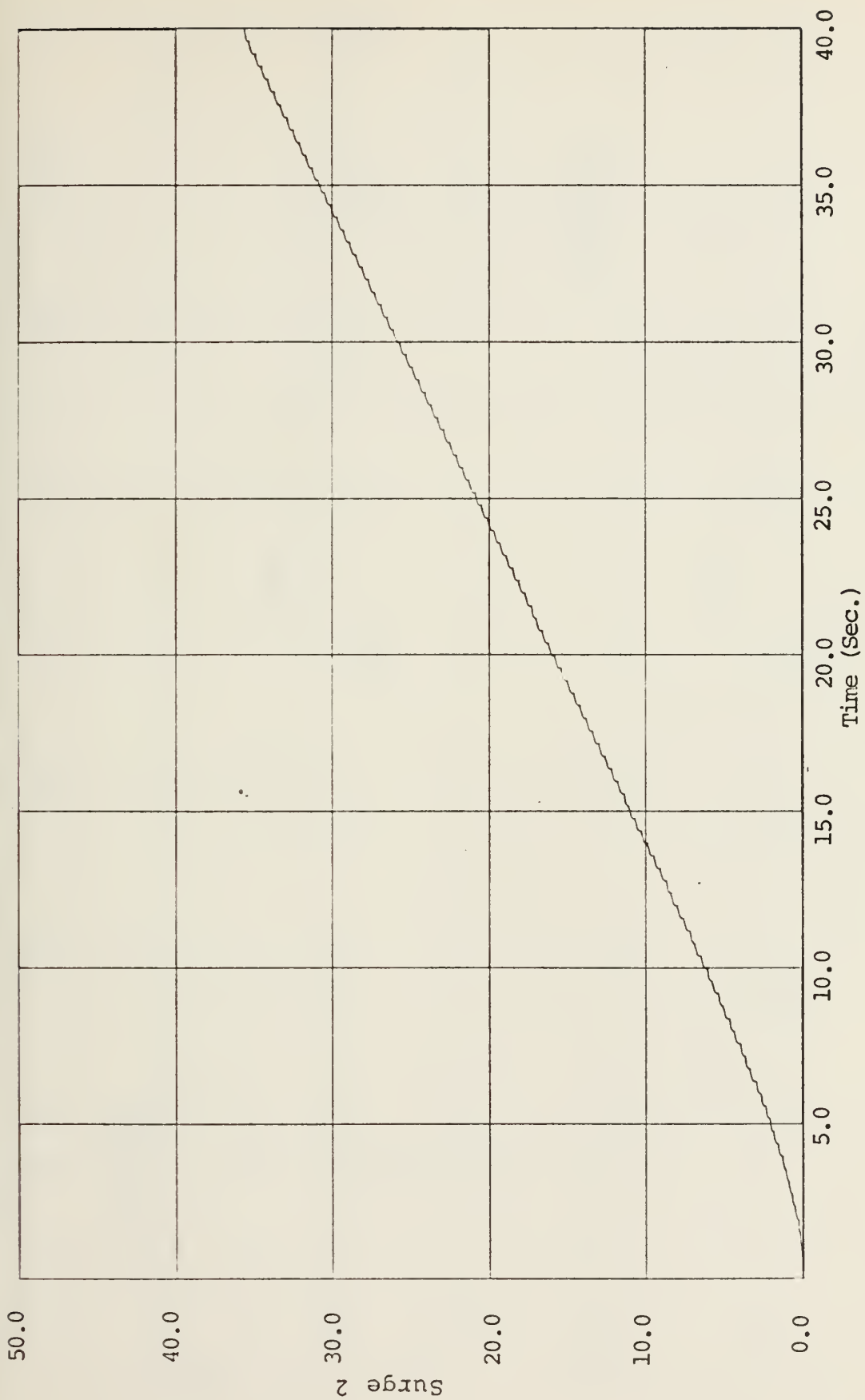


Figure 30. Surge 2 Vs. Time

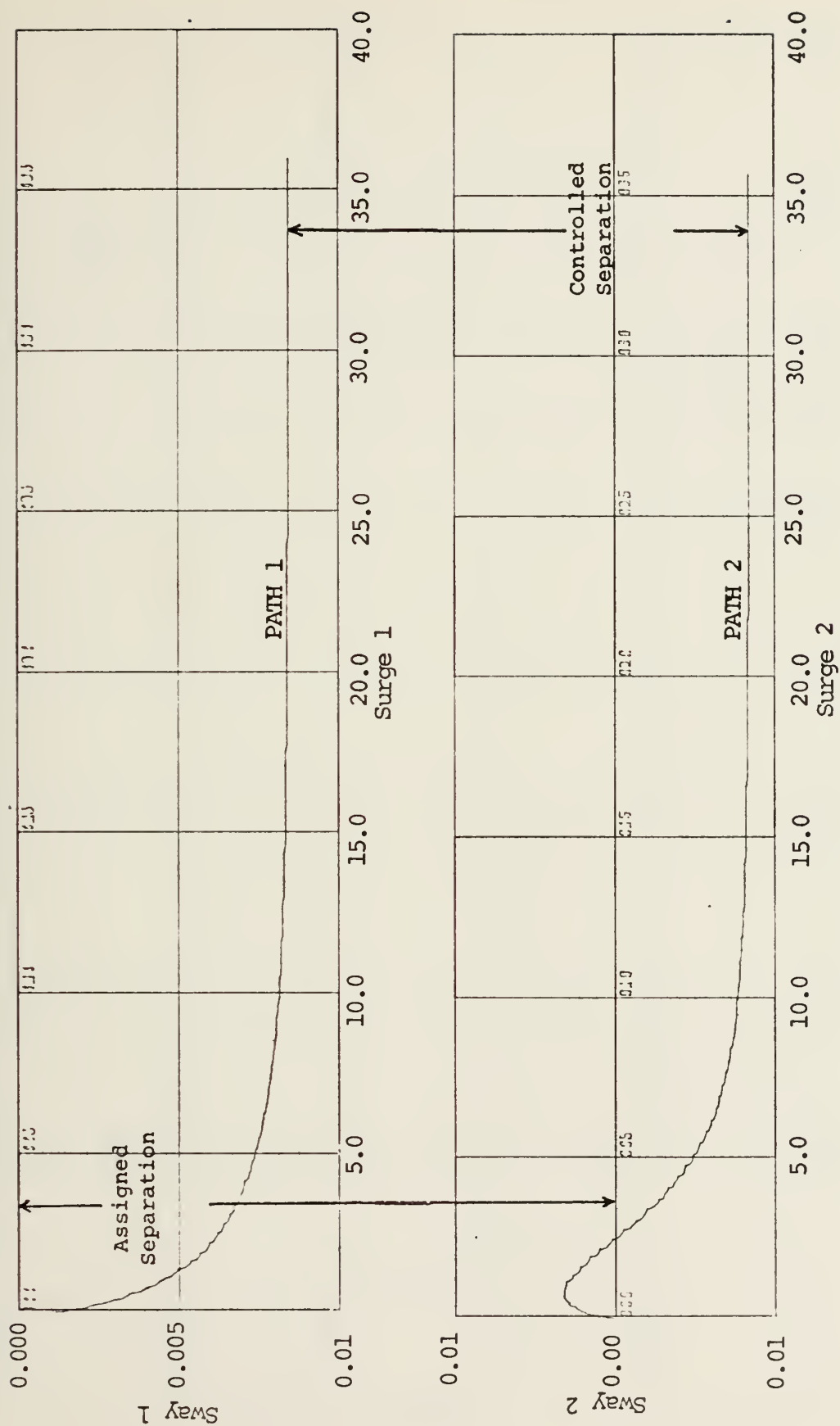


Figure 31. Trajectories of the Two Ships

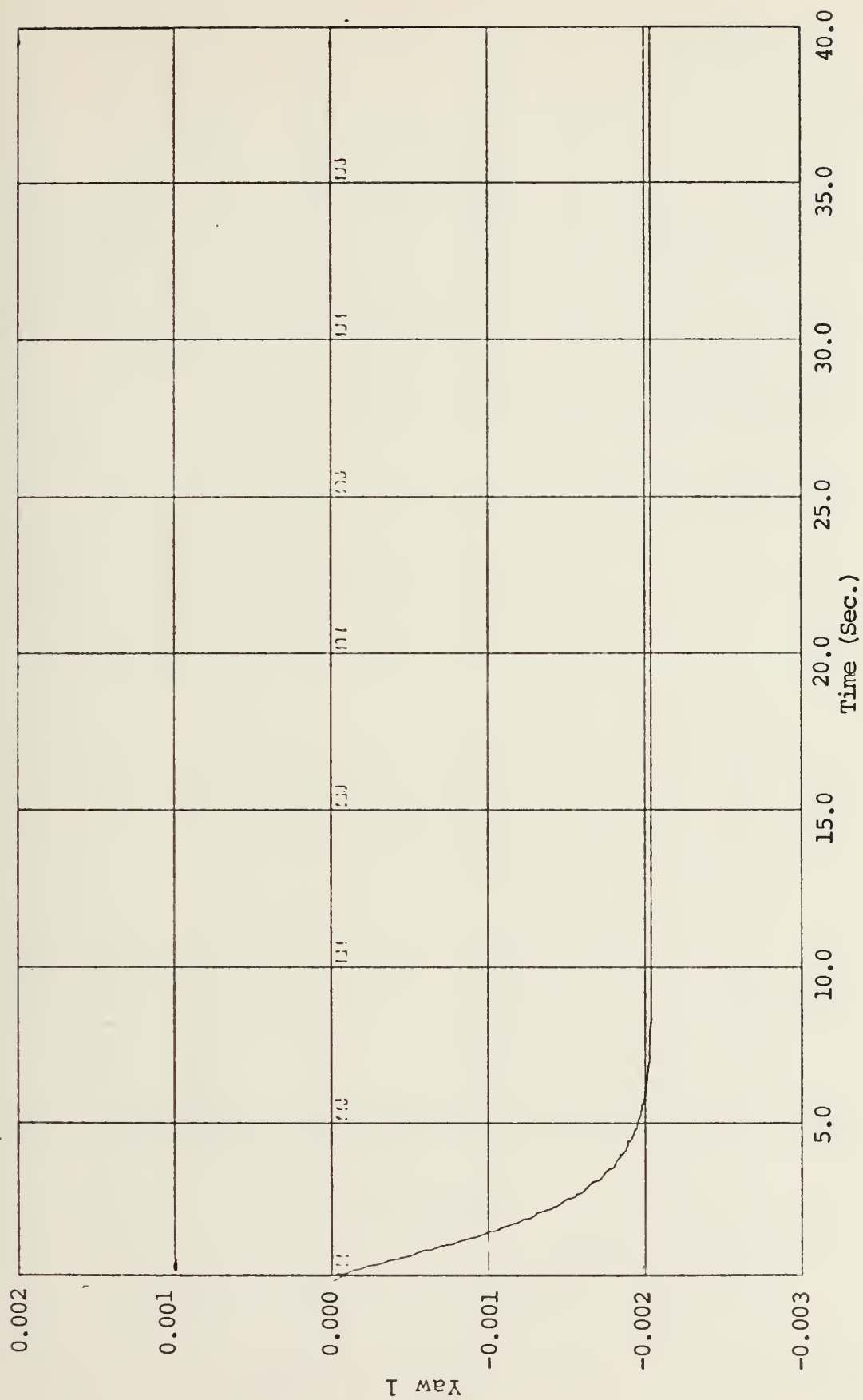


Figure 32. Yaw 1 Vs. Time

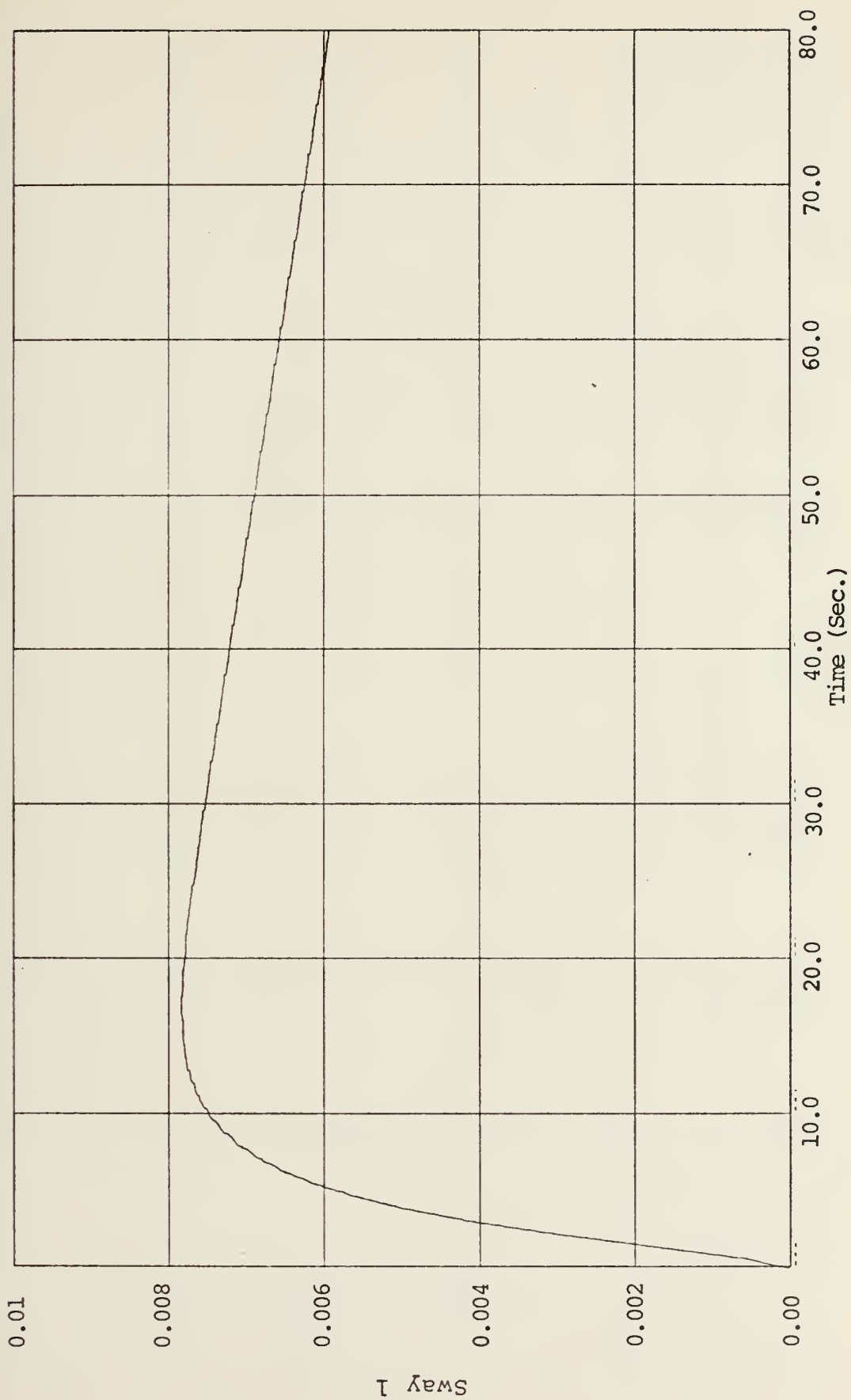


Figure 33. Sway 1 Vs. Time

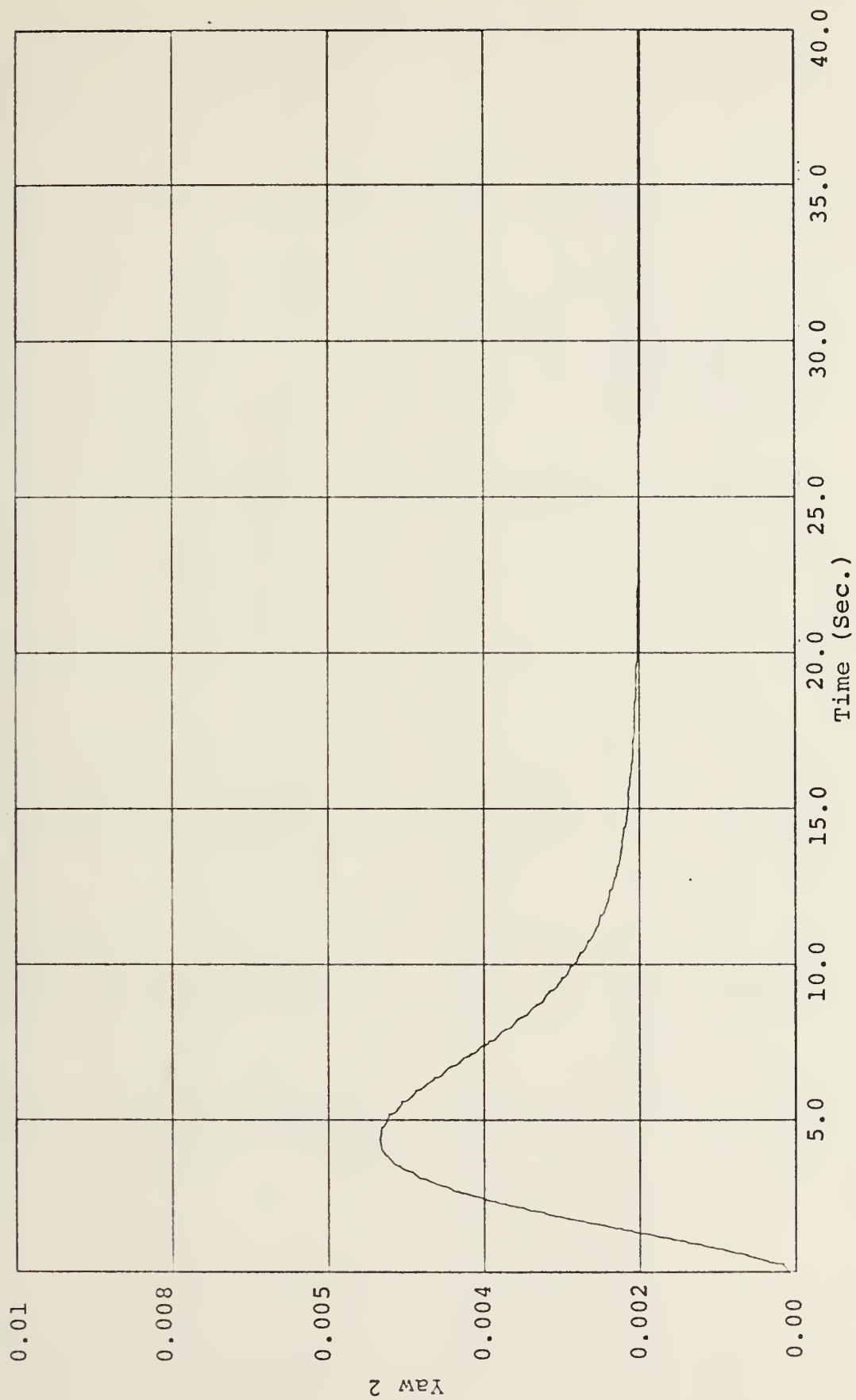


Figure 34. Yaw 2 Vs. Time



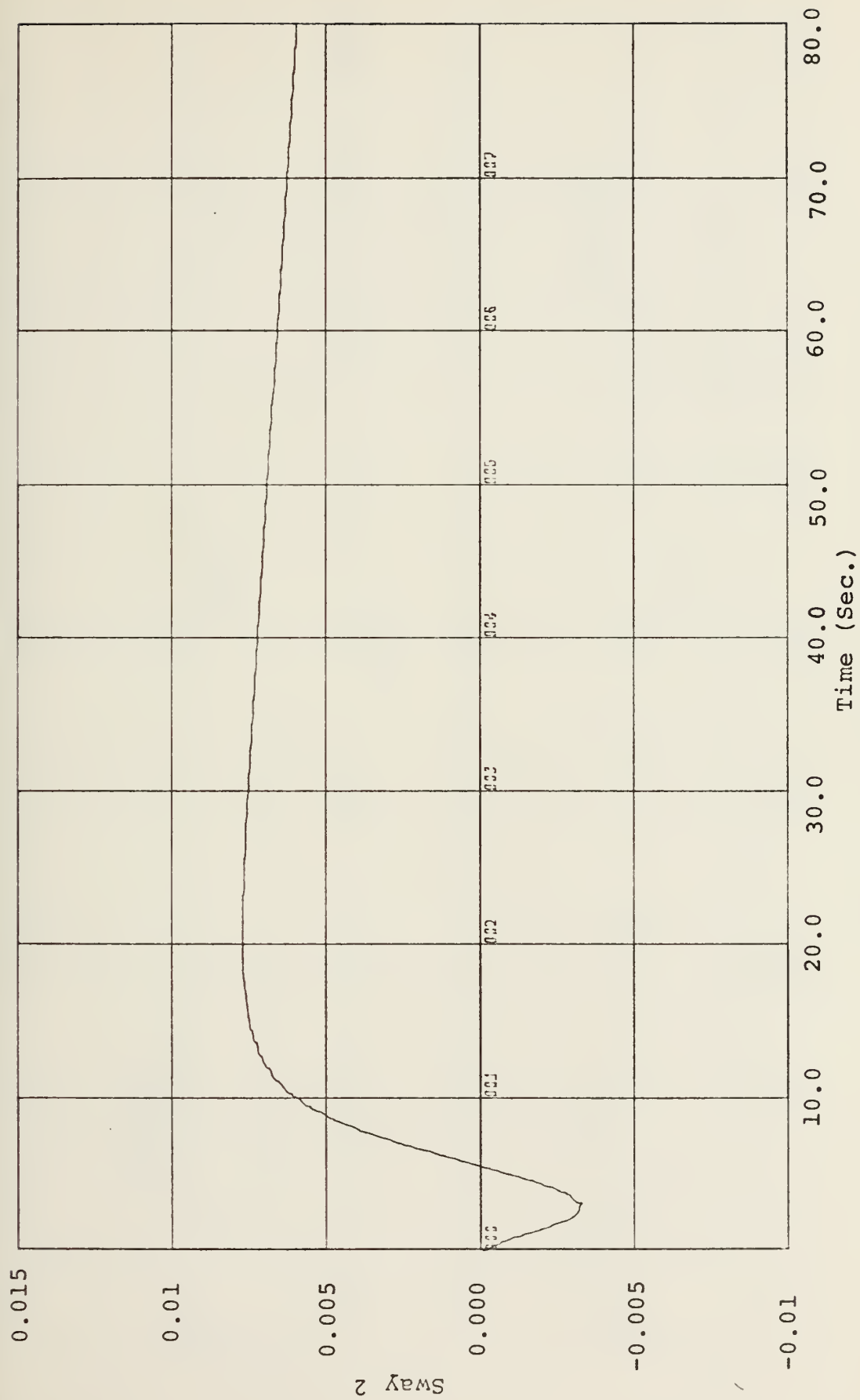


Figure 35. Sway 2 Vs. Time

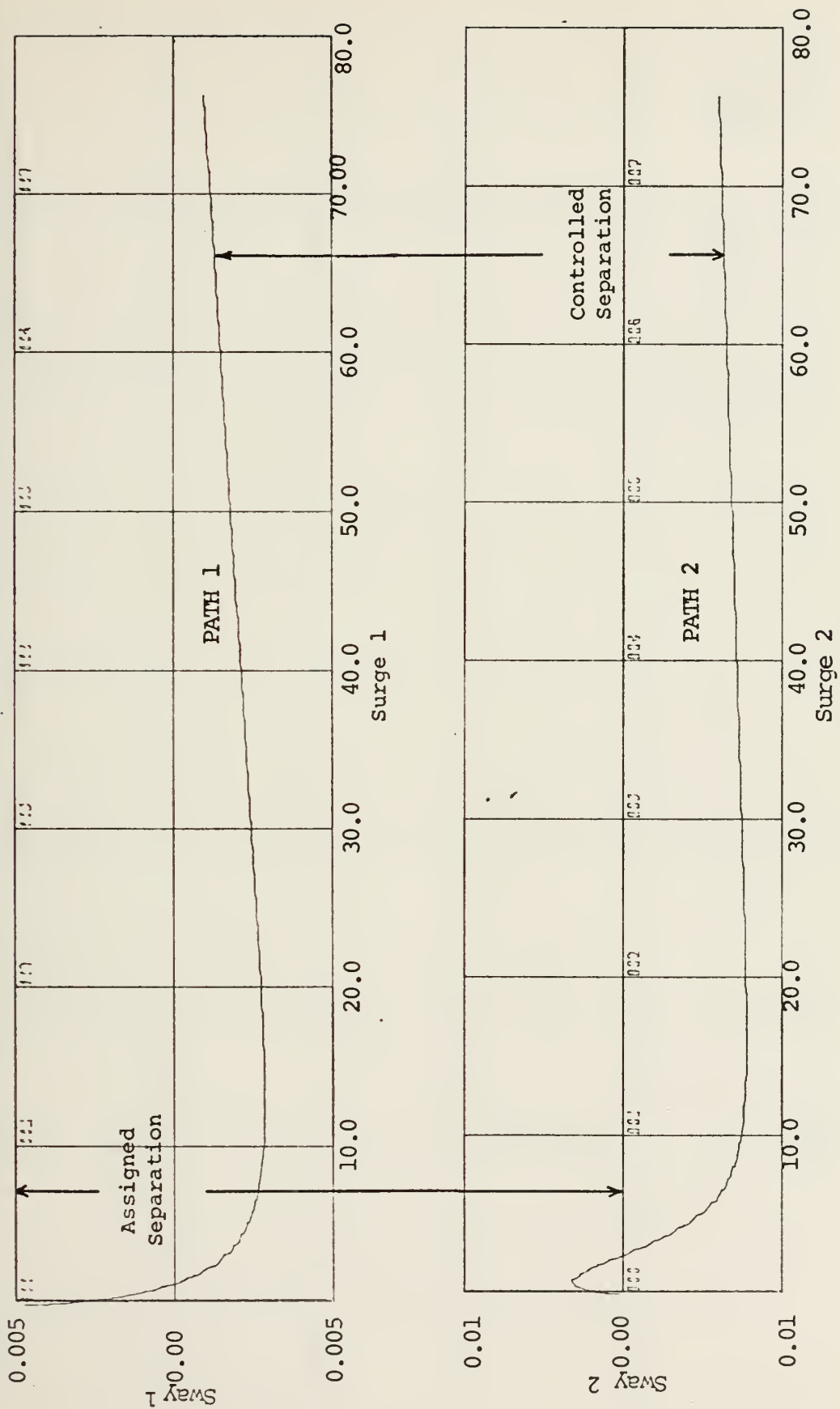


Figure 36. Trajectories of the Two Ships

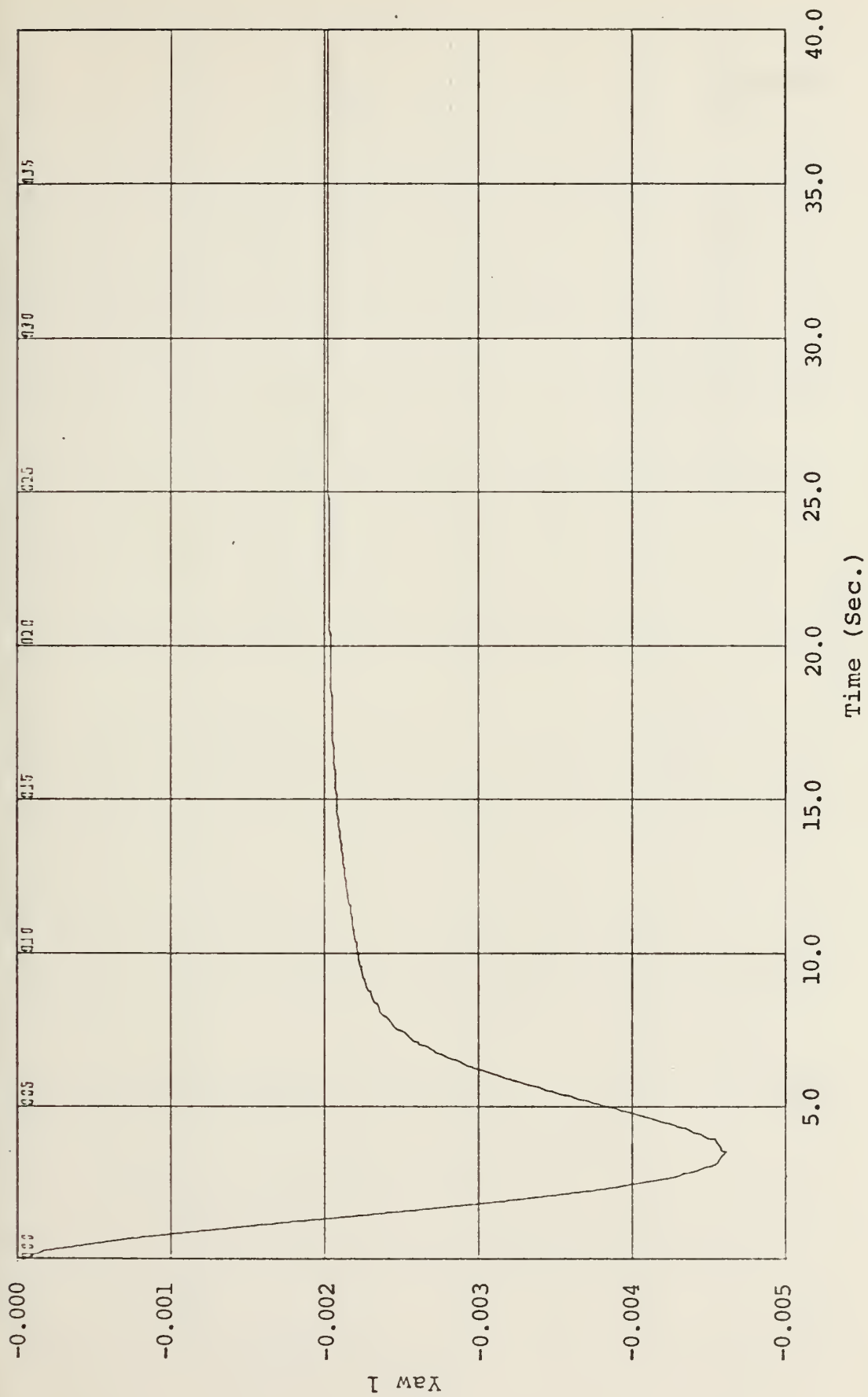


Figure 37. Yaw 1 Vs. Time

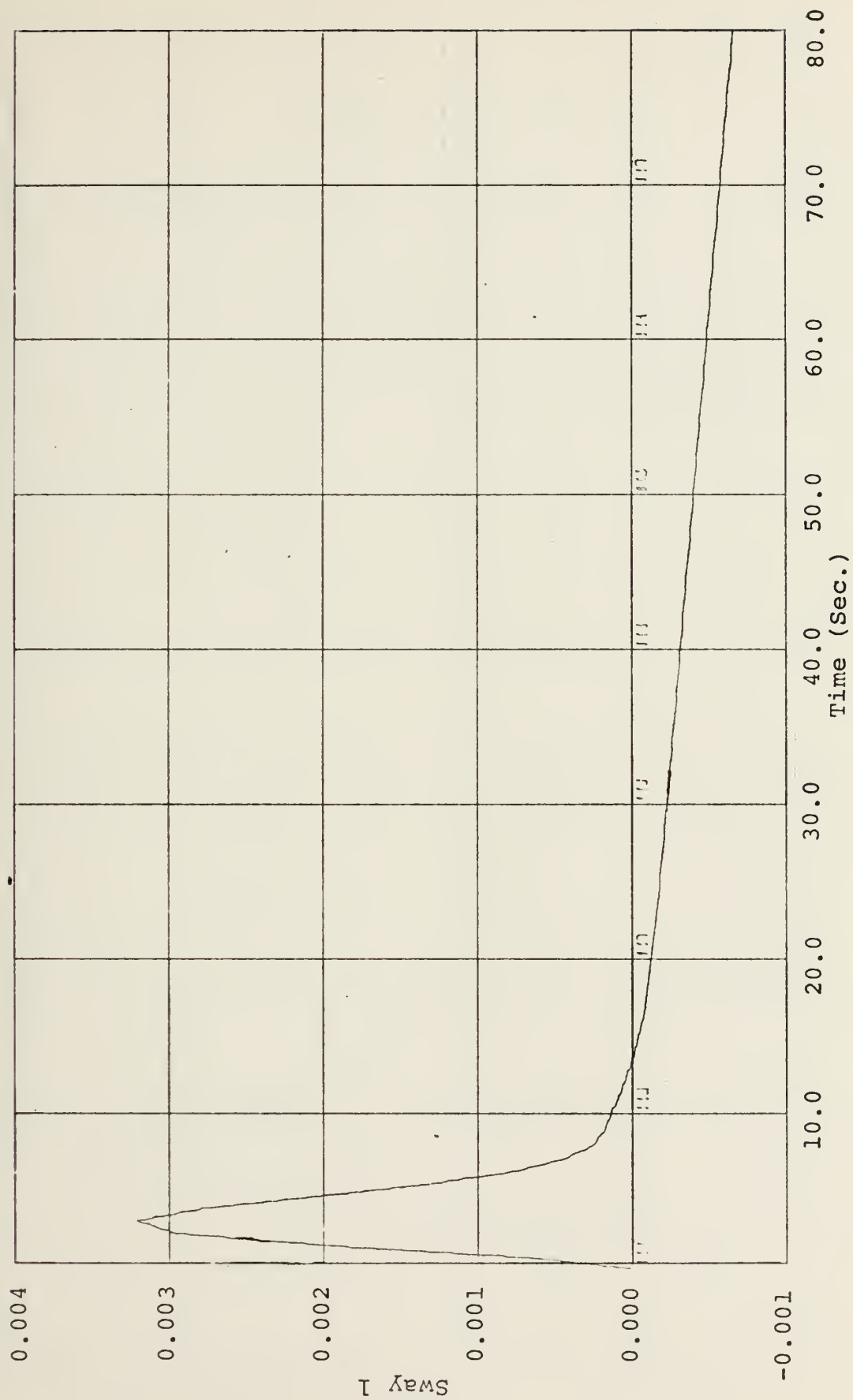


Figure 38. Sway 1 Vs. Time

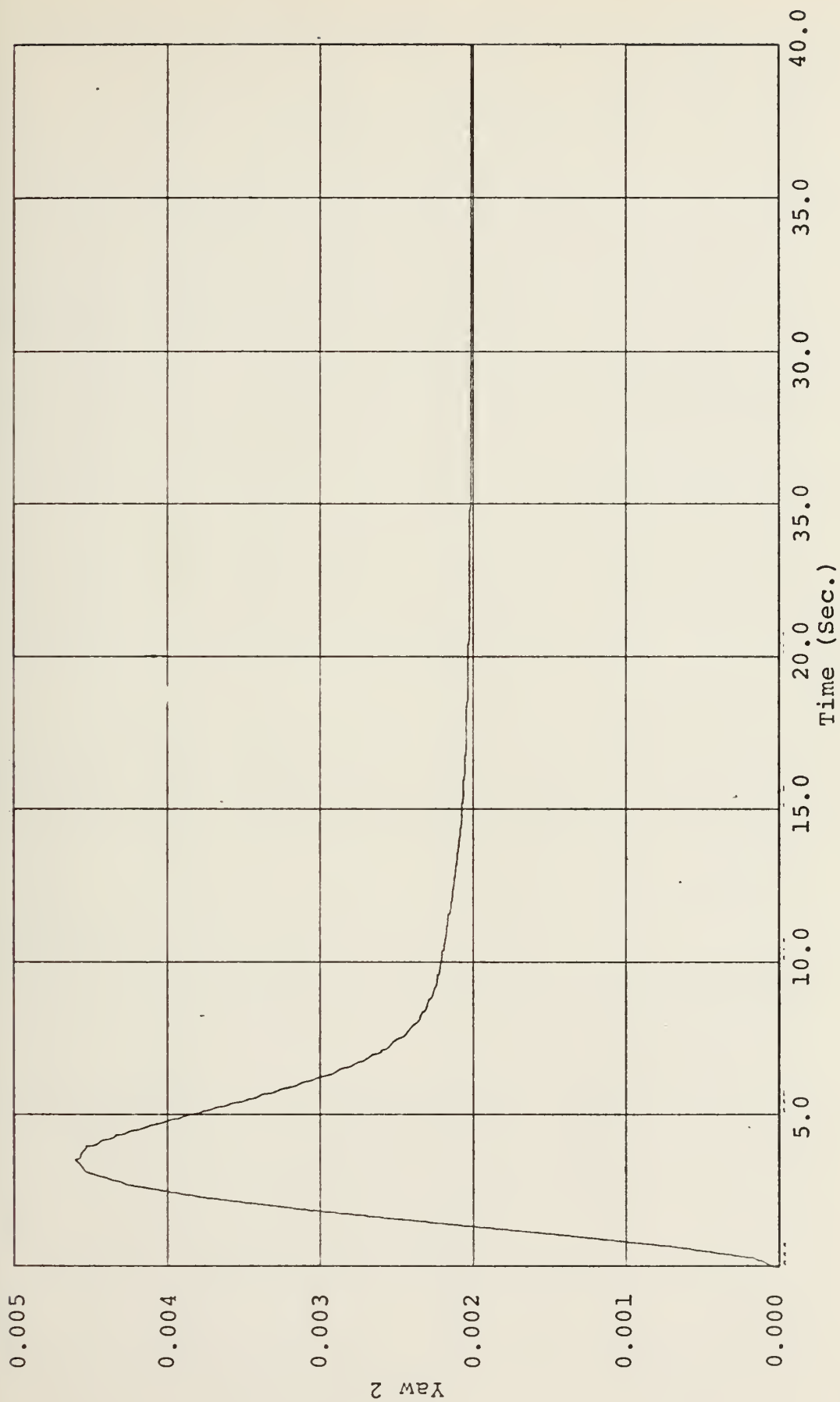


Figure 39. Yaw 2 Vs. Time

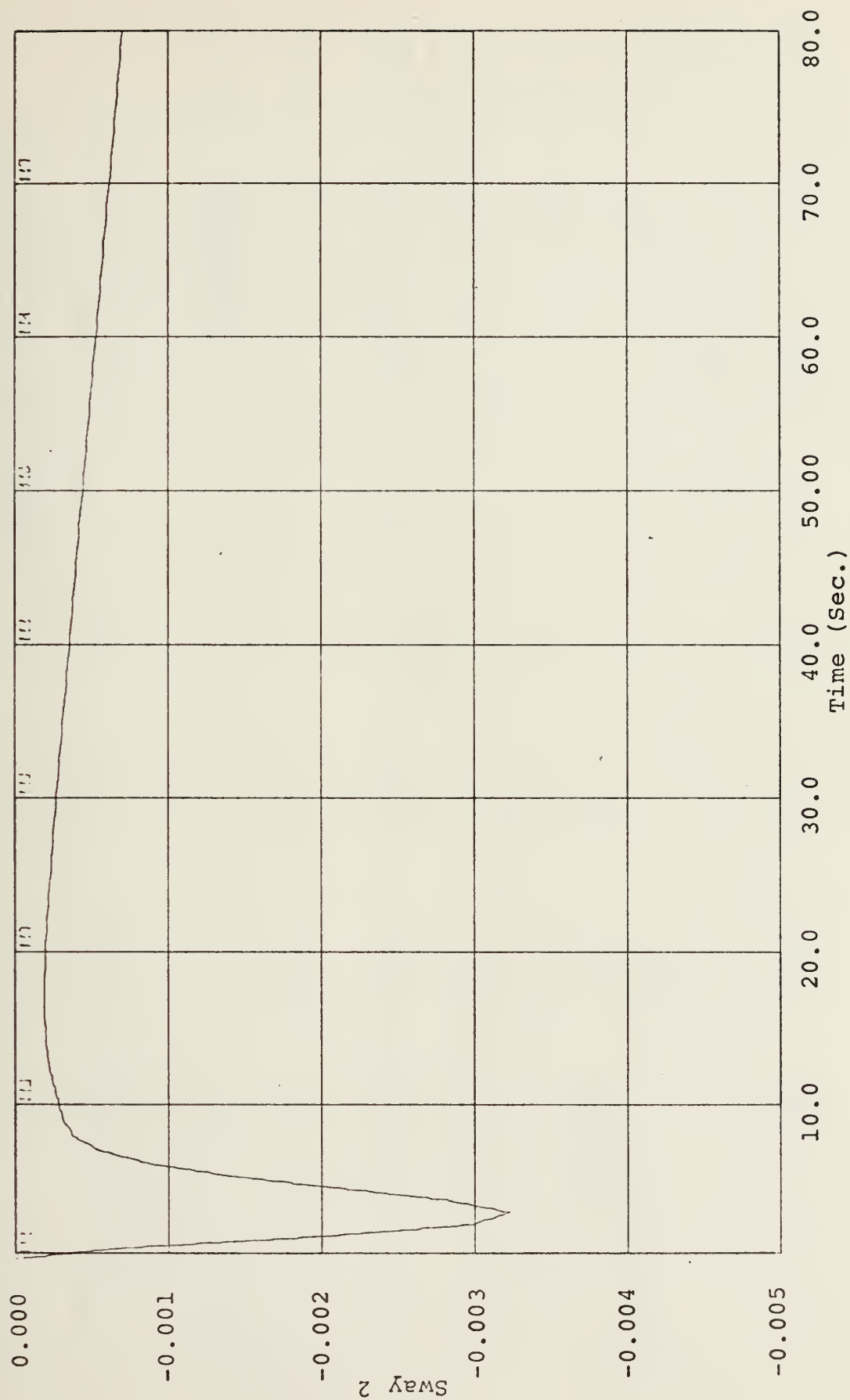


Figure 40. Sway 2 Vs. Time

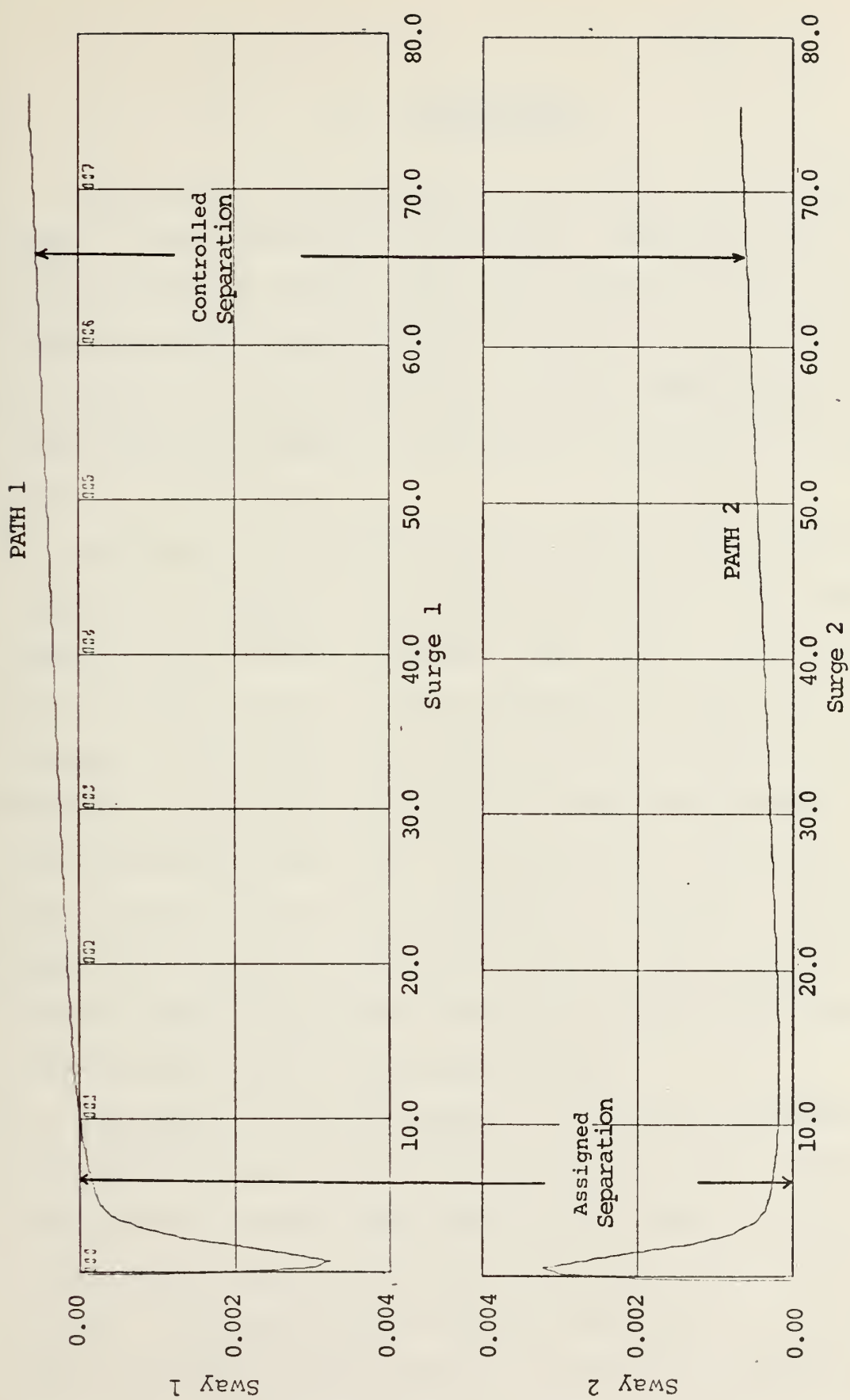


Figure 41. Trajectories of the Two Ships

VI. CONCLUSIONS

Comparison of the results of all three methods proposed leads to the conclusion that method I (complying with today's practiced Naval tactics, that the replenishing ship is only responsible for course or path keeping) is by far the best one. Of course the other two methods, in open sea, and when no other ships are present could still be considered feasible ones.

From Figs. 25 and 28 it is seen that if no corrective rudder (control) is applied, the interaction moments will cause the ships to yaw outwards, bringing their sterns toward each other. As the ships yaw the inwards hydrodynamic forces will come into action. If the exact and proper rudder is applied now, as in method I, this will counteract the interaction moment and bring the replenishing ship back to a position of equilibrium at a small angle of yaw to the direction of advance. In method I this angle of yaw is such that does not cause any change of course, since it produces the exact necessary lift force that balances the inward interaction force.

In method II, due to values of K_l and K_{Tl} different than the optimal ones, and in method III, due to the inclusion of the distance control loop, the applied rudder angle for the replenishing ship does not yield the proper equilibrium angle of yaw. In these methods although the moment of the force from the rudder can counteract the interaction moment, the rudder force is insufficient to balance the interaction force,

and the replenishing ship is therefore changing course, which the receiving ship is forced to follow.

COMPUTER PROGRAM I

```

*THIS PROGRAM SIMULATES THE DYNAMICS OF A SURFACE SHIP.
*THE EQUATIONS OF MOTION ARE TO BE IN THE FORM:
*(AAAS2+BAAS+GAA)A+(ABAS2+BBAS+GBA)B+(ACAS2+BCAS+GCA)C=
*KAI*D1+KA2*D2+NA
*(AABS2+BABS+GAB)A+(ABBS2+BBBS+GBB)B+(ACBS2+BCBS+GCB)C=
*KB1*D1+KB2*D2+NB
*(AACS2+BACS+GAC)A+(ABCS2+BBCS+GBC)B+(ACCS2+BCCS+GCC)C=
*KC1*D1+KC2*D2+NC
*A,B AND C ARE THE VARIABLES.
*D1,D2 ARE THE DEFLECTIONS OF THE RUDDER(S).
*NA,NB,NC ARE TERMS IN WHICH ANY EXTERNAL FORCES OR MOMENTS,
*OR NONLINEAR TERMS CAN BE INCLUDED.
*AAB,BAB,...,KAI...MUST BE DEFINED IN SECTION 2 AS FUNCTIONS
*OF THE HYDRODYNAMIC COEFFICIENTS,THE VALUES OF WHICH ARE
*INTRODUCED IN SECTION 1.
*NO ORDER IS REQUIRED.
*IN SECTION 3 THE VARIABLES ARE DEFINED,E.G. YAW=B.
*THE FIRST AND SECOND DERIVATIVES OF THE VARIABLES ARE:
*ADOT,BDOT,CBOT,ADDOT,BDDOT,CDDOT.
*IF THE REQUIRED SOLUTION INVOLVES TWO EQUATIONS,SET ACC=1.,
*THE SOLUTION WILL BE IN TERMS OF A AND B.
*UNDEFINED TERMS ARE SET AUTOMATICALLY TO ZERO,UP
*TO A MAXIMUM OF 85.
*SECTION 4 IS THE PROGRAMMED SIMULATION.
*SECTION 1 -COEFFICIENTS FOR A PARTICULAR SHIP
PARAM D1=0.1
PARAM M=0.0045
PARAM IZ=0.0003
PARAM NR=-0.0012
PARAM NRD=-0.0002
PARAM NV=-0.0012
PARAM NVD=-0.0001
PARAM KB1=-0.00084
PARAM YR=0.004
PARAM YRD=-0.0002
PARAM YV=-0.0063
PARAM YVD=-0.0025
PARAM KAI=0.0019
PARAM XU=-0.0012
PARAM XUD=-0.00036
PARAM KC1=-0.0011
*SECTION 2 -PARAMETERS CALCULATIONS
AAA=M-YVD
BAA=-YV
ABA=-YRD
BBA=M-YR
AAB=-NVD
BAB=-NV
ABB=IZ-NRD
BBB=-NR
ACC=M-XUD
BCC=-XU
NC=-XU
COFAA=ABB*ACC-ACB*ABC
COFAB=-(ABA*ACC-ACA*ABC)
COFAC=ABA*ACB-ABB*ACA
COFBA=-(AAB*ACC-ACB*AAC)
COFBB=AAA*ACC-ACA*AAC
COFBC=-(AAA*ACB-ACA*AAB)
COFCA=AAB*ABC-ABB*AAC
COFCB=-(AAA*ABC-ABA*AAC)
COFCC=AAA*ABB-ABA*AAB
S1=AAA*ABB*ACC+AAB*ABC*ACA+AAC*ACB*ABA
S2=ACA*ABB*AAC+ACB*ABC*AAA+ACC*AAB*ABA

```



```

DEL=S1-S2
*SECTION 3 -PHYSICAL DEFINITIONS
YAW=B
SWAY=Y
SURGE=X
*SECTION 4 -PROGRAMMED SIMULATION
YDOT=CDOT*SIN(YAW)+ADOT*COS(YAW)
Y=INTGRL(0.,YDOT)
XDOT=CDOT*COS(YAW)-ADOT*SIN(YAW)
X=INTGRL(0.,XDOT)
ADDOT=(COFAA*I1+COFAB*I2+COFAC*I3)/DEL
ADOT=INTGRL(0.,ADDOT)
A=INTGRL(0.,ADOT)
BDDOT=(COFBA*I1+COFBB*I2+COFBC*I3)/DEL
BDOT=INTGRL(0.,BDDOT)
B=INTGRL(0.,BDOT)
CDDOT=(COFCA*I1+COFCB*I2+COFCC*I3)/DEL
CDOT=INTGRL(0.,CDDOT)
C=INTGRL(0.,CDOT)
I1=-3AA*ADOT-3AA*A-BBA*BDOT-GBA*B-BCA*CDOT-GCA*C+IF1
I2=-BAB*ADOT-GAB*A-BBB*BDOT-GBB*B-BCB*CDOT-GCB*C+IF2
I3=-BAC*ADOT-SAC*A-BBC*BDOT-GBC*B-BCC*CDOT-GCC*C+IF3
IF1=KA1*D1+KA2*D2+NA
IF2=KB1*D1+KB2*D2+NB
IF3=KC1*D1+KC2*D2+NC
*SECTION 5 -OUTPUT CHARACTERISTICS
PREPAR TIME,YAW
PRTPLOT YAW
LABEL S3-LAT. DYNAMICS
TIMER FINTIM=40.0,DELT=0.02,PRDEL=0.2
END
STOP
ENDJOB

```


COMPUTER PROGRAM II

```

*THIS PROGRAM SIMULATES THE DYNAMICS OF A SURFACE SHIP.
*THE EQUATIONS OF MOTION ARE TO BE IN THE FORM:
*(AAAS3+BAAS2+GAAS+JAA)A+(ABAS3+BBAS2+GBAS+DBA)B+
*(ACAS3+BCAS2+GCAS+DCA)C=KA1*D1+KA2*D2+NA
*(AABS3+BABS2+GABS+DAB)A+(ABBS3+BBBS2+GBBS+DBB)B+
*(ACBS3+BCBS2+GCBS+DCB)C=KB1*D1+KB2*D2+NB
*(AACS3+BACS2+GACS+DAC)A+(ABCS3+BBCS2+GBCS+DBC)B+
*(ACCS3+BCCS2+GCCS+DCC)C=KC1*D1+KC2*D2+NC
*A,B AND C ARE THE VARIABLES.
*D1,D2 ARE THE DEFLECTIONS OF THE RUDDER(S).
*NA,NB,NC ARE TERMS IN WHICH ANY EXTERNAL FORCES OR MOMENTS,
*OR NONLINEAR TERMS CAN BE INCLUDED.
*AAB,BAB,...,KA1...MUST BE DEFINED IN SECTION 2 AS FUNCTIONS
*OF THE HYDRODYNAMIC COEFFICIENTS,THE VALUES OF WHICH ARE
*INTRODUCED IN SECTION 1.
*NO ORDER IS REQUIRED.
*IN SECTION 3 THE VARIABLES ARE DEFINED,E.G. YAW=B.
*THE FIRST AND SECOND DERIVATIVES OF THE VARIABLES ARE:
*ADOT,BDOT,CDOT,ACDOT,BDDOT,CDDOT.
*IF THE REQUIRED SOLUTION INVOLVES TWO EQUATIONS,SET ACC=1.,
*THE SOLUTION WILL BE IN TERMS OF A AND B.
*UNDEFINED TERMS ARE SET AUTOMATICALLY TO ZERO,UP
*TO A MAXIMUM OF 85.
*SECTION 4 IS THE PROGRAMMED SIMULATION.
*SECTION 1 -COEFFICIENTS FOR A PARTICULAR SHIP
PARAM K=0.78137
PARAM KT=0.88935
PARAM M=0.0045
PARAM IZ=0.0003
PARAM NR=-0.0012
PARAM NRD=-0.0002
PARAM NV=-0.0012
PARAM NVD=-0.0001
PARAM KB1=-0.00084
PARAM YR=0.004
PARAM YRD=-0.0002
PARAM YV=-0.0063
PARAM YVD=-0.0025
PARAM KA1=0.0019
PARAM XU=-0.0012
PARAM XUD=-0.00036
PARAM KC1=-0.0011
*SECTION 2 -PARAMETERS CALCULATIONS
AAA=M-YVD
BAA=-YV+10.0*(M-YVD)
ABA=-YRD
EBA=M-YR-10.0*KA1*KT
AAB=-NVD
BAB=-NV-10.0*VVD
ABB=IZ-NRD
BBB=-NR+10.0*(IZ-NRD)
BCC=-XU+10.0*(M-XUD)
ACC=M-XUD
NC=-10.0*XU
GAA=-10.0*YV
GBA=10.0*(M-YR)-10.0*KA1*KT
DBA=-10.0*KA1*K
GAB=-10.0*NV
GSB=-10.0*NR-10.0*KB1*KT
DBB=-10.0*KB1*K
GBC=-10.0*KC1*KT
DBC=-10.0*KC1*K
GCC=-10.0*XU
COFAA=ABB*ACC-ACB*ABC

```



```

COFAB=-(ABA*ACC-ACA*ABC)
COFAC=ABA*ACB-ABB*ACA
COFBA=-(AAB*ACC-ACB*AAC)
COFBB=AAA*ACC-ACA*AAC
COFBC=-(AAA*ACB-ACA*AAB)
COFCA=AAB*ABC-ABB*AAC
COFCB=-(AAA*ABC-ABA*AAC)
COFCC=AAA*ABB-ABA*AAB
S1=AAA*ABB*ACC+AAB*ABC*ACA+AAC*ACB*ABA
S2=ACA*ABB*AAC+ACB*ABC*AAA+ACC*AAB*ABA
DEL=S1-S2
*SECTION 3 -PHYSICAL DEFINITIONS
YAW=B
SWAY=Y
SURGE=X
*SECTION 4 -PROGRAMMED SIMULATION
ADDDOT=(COFAA*I1+COFAB*I2+COFAC*I3)/DEL
ADDOT=INTGRL(0.,ADDDOT)
ADOT=INTGRL(0.,ADDOT)
A=INTGRL(0.,ADOT)
BDDDOT=(COFBA*I1+COFBB*I2+COFBC*I3)/DEL
BDDDOT=INTGRL(0.,BDDDOT)
BDOT=INTGRL(0.1,BDDDOT)
B=INTGRL(0.2,BDOT)
CDDDOT=(COFCA*I1+COFCB*I2+COFCC*I3)/DEL
CDDDOT=INTGRL(0.,CDDDOT)
CDOT=INTGRL(0.,CDDDOT)
C=INTGRL(0.,CDOT)
I1=-3AA*ADDDOT-GAA*ADOT-BBA*BDDDOT-GBA*BDOT+IF1-DAA*A-DBA*B
I2=-3AB*ADDDOT-GAB*ADOT-BBB*BDDDOT-GBB*BDOT+IF2-DAB*A-DBB*B
I3=-3BC*BDDDOT-GBC*BDOT-BCC*CDDDOT-GCC*CDOT+IF3-DCC*C-DBC*B
IF1=0.0
IF2=0.0
IF3=NC
*SECTION 5 -OUTPUT CHARACTERISTICS
PREPAR TIME,YAW
PRTPLOT YAW
LABEL S3-LAT. DYNAMICS
TIMER FINTIM=40.0,DELT=0.02,PRDEL=0.2
END
STOP
ENDJOB

```


COMPUTER PROGRAM III

```

*THIS PROGRAM SIMULATES THE DYNAMICS OF A SURFACE SHIP.
*THE EQUATIONS OF MOTION ARE TO BE IN THE FORM:
*(AAAS3+BAAS2+GAAS+DAA)A+(ABAS3+B3AS2+GBAS+DBA)B+
*(ACAS3+BCAS2+GCAS+DCA)C=KA1*D1+KA2*D2+NA
*(AABS3+BABS2+GABS+DAB)A+(ABBS3+BBBS2+GBBS+DBB)B+
*(ACBS3+BCBS2+GCBS+DCB)C=KB1*D1+KB2*D2+NB
*(AACS3+BACS2+GACS+DAC)A+(ABCS3+BBCS2+GBCS+DBC)B+
*(ACCS3+BCCS2+GCCS+DCC)C=KC1*D1+KC2*D2+NC
*A,B AND C ARE THE VARIABLES.
*D1,D2 ARE THE DEFLECTIONS OF THE RUDDER(S).
*NA,NB,NC ARE TERMS IN WHICH ANY EXTERNAL FORCES OR MOMENTS,
*OR NONLINEAR TERMS CAN BE INCLUDED.
*AAB,BAB,...,KA1...MUST BE DEFINED IN SECTION 2 AS FUNCTIONS
*OF THE HYDRODYNAMIC COEFFICIENTS,THE VALUES OF WHICH ARE
*INTRODUCED IN SECTION 1.
*NO ORDER IS REQUIRED.
*IN SECTION 3 THE VARIABLES ARE DEFINED,E.G. YAW=B.
*THE FIRST AND SECOND DERIVATIVES OF THE VARIABLES ARE:
*ADOT,BDOT,CDOT,ADDOT,BDDOT,CDDOT.
*IF THE REQUIRED SOLUTION INVOLVES TWO EQUATIONS,SET ACC=1.,
*THE SOLUTION WILL BE IN TERMS OF A AND B.
*UNDEFINED TERMS ARE SET AUTOMATICALLY TO ZERO,UP
*TO A MAXIMUM OF 85.
*SECTION 4 IS THE PROGRAMMED SIMULATION.
*SECTION 1 -COEFFICIENTS FOR A PARTICULAR SHIP
PARAM D=0.1
PARAM K=0.0225
PARAM KT=0.25942
PARAM M=0.0045
PARAM IZ=0.0003
PARAM NR=-0.0012
PARAM NRD=-0.0002
PARAM NV=-0.0012
PARAM NVD=-0.0001
PARAM KB1=-0.00084
PARAM YR=0.004
PARAM YRD=-0.0002
PARAM YV=-0.0063
PARAM YVD=-0.0025
PARAM KA1=0.0019
PARAM XU=-0.0012
PARAM XUD=-0.00036
PARAM KC1=-0.0011
*SECTION 2 -PARAMETERS CALCULATIONS
AAA=M-YVD
BAA=-YV+10.0*(M-YVD)
ABA=-YRD
BBA=M-YR-10.0*YRD
AAB=-NVD
BAB=-NV-10.0*NVD
ABB=IZ-NRD
BBB=-NR+10.0*(IZ-NRD)
BCC=-XU+10.0*(M-XUD)
ACC=M-XUD
NC=-10.0*XU
GAA=-10.0*YV
GBA=10.0*(M-YR)
GAB=-10.0*NV
GBB=-10.0*NR
GCC=-10.0*XU
COFAA=ABB*ACC-ACB*ABC
COFAB=-(ABA*ACC-ACA*ABC)
COFAC=ABA*ACB-ABB*ACA
COFBA=-(AAB*ACC-ACB*AAC)

```



```

COFBB=AAA*ACC-ACA*AAC
COFBC=-(AAA*ACB-ACA*AAB)
COFCA=AAB*ABC-ABB*AAC
COFCB=-(AAA*ABC-ABA*AAC)
COFCC=AAA*ABB-ABA*AAB
S1=AAA*ABB*ACC+AAB*ABC*ACA+AAC*ACB*ABA
S2=ACA*ABB*AAC+ACB*ABC*AAA+ACC*AAB*ABA
DEL=S1-S2
*SECTION 3 -PHYSICAL DEFINITIONS
YAW=B
SWAY=Y
SURGE=X
*SECTION 4 -PROGRAMMED SIMULATION
ADDDOT=(COFAA*I1+COFAB*I2+COFAC*I3)/DEL
ADDDOT=INTGRL(0.,ADDDOT)
ADOT=INTGRL(0.01,ADDDOT)
A=INTGRL(0.,ADOT)
BDDDOT=(COFBA*I1+COFBB*I2+COFBC*I3)/DEL
BDDDOT=INTGRL(0.,BDDDOT)
BDOT=INTGRL(0.,BDDDOT)
B=INTGRL(0.,BDOT)
CDDDOT=(COFCA*I1+COFCB*I2+COFCC*I3)/DEL
CDDDOT=INTGRL(0.,CDDDOT)
CDOT=INTGRL(0.,CDDDOT)
C=INTGRL(0.,CDOT)
I1=-3AA*ADDDOT-GAA*ADOT-BBA*BDDDOT-GBA*BDOT+IF1-DAA*A-DBA*B
I2=-BAB*ADDDOT-GAB*ADOT-BBB*BDDDOT-GBB*BDOT+IF2-DAB*A-DBB*B
I3=-BBC*BDDDOT-GBC*BDOT-BCC*CDDDOT-GCC*CDOT+IF3-DCC*C-DBC*B
IF1=10.0*KAI*DD
IF2=10.0*KBI*DD
IF3=NC+10.0*KCI*DD
DD=K*Y+KT*YDOT
YDOT=CDOT*SIN(YAW)+ADOT*COS(YAW)
Y=INTGRL(0.1,YDOT)
*SECTION 5 -OUTPUT CHARACTERISTICS
PREPAR TIME,YAW
PRTPLOT YAW
LABEL S3-LAT. DYNAMICS
TIMER FINTIM=40.0,DELT=0.02,PRDEL=0.2
END
STOP
ENDJOB

```


COMPUTER PROGRAM IV

```

*THIS PROGRAM SIMULATES THE DYNAMICS OF A SURFACE SHIP.
*THE EQUATIONS OF MOTION ARE TO BE IN THE FORM:
*(AAAS3+BAAS2+GAAS+JAA)A+(ABAS3+BBAS2+GBAS+DBA)B+
*(ACAS3+BCAS2+GCAS+DCA)C=KA1*D1+KA2*D2+NA
*(AABS3+BABS2+GABS+DAB)A+(ABBS3+BBBS2+GBBS+DBB)B+
*(ACBS3+BCBS2+GCBs+DCB)C=KB1*D1+KB2*D2+NB
*(AACS3+BACS2+GACS+DAC)A+(ABCS3+BBCS2+GBCS+DBC)B+
*(ACCS3+BCCS2+GCCS+DCC)C=KC1*D1+KC2*D2+NC
*A,B AND C ARE THE VARIABLES.
*D1,D2 ARE THE DEFLECTIONS OF THE RUDDER(S).
*NA,NB,NC ARE TERMS IN WHICH ANY EXTERNAL FORCES OR MOMENTS,
*OR NONLINEAR TERMS CAN BE INCLUDED.
*AA3,BAB...,KA1...MUST BE DEFINED IN SECTION 2 AS FUNCTIONS
*OF THE HYDRODYNAMIC COEFFICIENTS,THE VALUES OF WHICH ARE
*INTRODUCED IN SECTION 1.
*NO ORDER IS REQUIRED.
*IN SECTION 3 THE VARIABLES ARE DEFINED,E.G. YAW=B.
*THE FIRST AND SECOND DERIVATIVES OF THE VARIABLES ARE:
*ADOT,BDOT,CDOT,ADDOT,BDDOT,CDDOT.
*IF THE REQUIRED SOLUTION INVOLVES TWO EQUATIONS,SET ACC=1.,
*THE SOLUTION WILL BE IN TERMS OF A AND B.
*UNDEFINED TERMS ARE SET AUTOMATICALLY TO ZERO,UP
*TO A MAXIMUM OF 85.
*SECTION 4 IS THE PROGRAMMED SIMULATION.
*SECTION 1 -COEFFICIENTS FOR A PARTICULAR SHIP
PARAM D=0.1
PARAM K=0.0225
PARAM KT=0.25942
PARAM M=0.0045
PARAM IZ=0.0003
PARAM NR=-0.0012
PARAM NRD=-0.0002
PARAM NV=-0.0012
PARAM NVD=-0.0001
PARAM KB1=-0.00084
PARAM YR=0.004
PARAM YRD=-0.0002
PARAM YV=-0.0063
PARAM YVD=-0.0025
PARAM KA1=0.0019
PARAM XU=-0.0012
PARAM XUD=-0.00036
PARAM KC1=-0.0011
*SECTION 2 -PARAMETERS CALCULATIONS
AAA=M-YVD
BAA=-YV+10.0*(M-YVD)
ABA=-YRD
BBA=M-YR-10.0*YRD
AAB=-NVD
BAB=-NV-10.0*NVD
ABB=IZ-NRD
BBB=-NR+10.0*(IZ-NRD)
BCC=-XU+10.0*(M-XUD)
ACC=M-XUD
NC=-10.0*XU
GAA=-10.0*YV
GBA=10.0*(M-YR)
GAB=-10.0*Nv
GBB=-10.0*NR
GCC=-10.0*XU
COFAA=ABB*ACC-ACB*ABC
COFAB=-(ABA*ACC-ACA*ABC)
COFAC=ABA*ACB-ABB*ACA
COFBA=-(AAB*ACC-ACB*AAC)

```



```

COFBB=AAA*ACC-ACA*AAC
COFBC=-(AAA*ACB-ACA*AAB)
COFCA=AAB*ABC-ABB*AAC
COFCB=-(AAA*ABC-ABA*AAC)
COFCC=AAA*ABB-ABA*AAB
S1=AAA*AAB*ACC+AAB*ABC*ACA+AAC*ACB*ABA
S2=ACA*ABB*AAC+ACB*ABC*AAA+ACC*AAB*ABA
DEL=S1-S2
*SECTION 3 -PHYSICAL DEFINITIONS
YAW=B
SWAY=Y
SURGE=X
*SECTION 4 -PROGRAMMED SIMULATION
ADDDOT=(COFAA*I1+COFAB*I2+COFAC*I3)/DEL
ADDOT=INTGRL(0.,ADDDOT)
ADOT=INTGRL(0.,ADDOT)
A=INTGRL(0.,ADOT)
BDDDOT=(COFBA*I1+COFBB*I2+COFBC*I3)/DEL
BDDDOT=INTGRL(0.,BDDDOT)
BDDOT=INTGRL(0.,BDDDOT)
B=INTGRL(0.,BDDOT)
CDDDOT=(COFCA*I1+COFCB*I2+COFCC*I3)/DEL
CDDDOT=INTGRL(0.,CDDDOT)
CDDOT=INTGRL(0.,CDDDOT)
C=INTGRL(0.,CDDOT)
I1=-3AA*ADDDOT-GAA*ADOT-BBA*BDDDOT-GBA*BDDOT+IF1-DAA*A-DBA*B
I2=-BAB*ADDDOT-GAB*ADOT-BBB*BDDDOT-GBB*BDDOT+IF2-DAB*A-DBB*B
I3=-BBC*BDDDOT-GBC*BDDOT-BCC*CDDDOT-GCC*CDDOT+IF3-DCC*C-DBC*B
IF1=10.0*KAI*DD
IF2=10.0*KBI*DD
IF3=NC+10.0*KCI*DD
DD=K*(Y-D)+KT*YDOT
YDOT=CDDOT*SIN(YAW)+ADOT*COS(YAW)
Y=INTGRL(0.,YDOT)
*SECTION 5 -OUTPUT CHARACTERISTICS
PREPAR TIME,SWAY,SURGE,YAW
PRTPLOT SWAY
LABEL S3-LAT. DYNAMICS
TIMER FINTIM=40.0,DELT=0.02,PRDEL=0.2
END
STOP
ENDJOB

```


COMPUTER PROGRAM V

```

*THIS PROGRAM SIMULATES THE DYNAMICS OF TWO SURFACE SHIPS.
*THE EQUATIONS OF MOTION FOR EACH ARE TO BE IN THE FORM:
*(AAAS3+BAAS2+GAAS+DAA)A+(ABAS3+BBAS2+GBAS+DBA)B+
*(ACAS3+BCAS2+GCAS+DCA)C=KA1*D1+KA2*D2+NA
*(AABS3+BABS2+GABS+DAB)A+(ABBS3+BBBS2+GBBS+DBB)B+
*(ACBS3+BCBS2+GCBS+DCB)C=KB1*D1+KB2*D2+NB
*(AACS3+BACS2+GACS+DAC)A+(ABCS3+BBCS2+GBCS+DBC)B+
*(ACCS3+BCCS2+GCCS+DCC)C=KC1*D1+KC2*D2+NC
*A,B AND C ARE THE VARIABLES.
*D1,D2 ARE THE DEFLECTIONS OF THE RUDDER(S).
*NA,NB,NC ARE TERMS IN WHICH ANY EXTERNAL FORCES OR MOMENTS,
*OR NONLINEAR TERMS CAN BE INCLUDED.
*AAB,BAB,...,KA1...MUST BE DEFINED IN SECTION 2 AS FUNCTIONS
*OF THE HYDRODYNAMIC COEFFICIENTS,THE VALUES OF WHICH ARE
*INTRODUCED IN SECTION 1.
*NO ORDER IS REQUIRED.
*IN SECTION 3 THE VARIABLES ARE DEFINED,E.G. YAW=B.
*THE FIRST AND SECOND DERIVATIVES OF THE VARIABLES ARE:
*ADOT,BDOT,CDOT,ADDOT,BDDOT,CDDOT.
*IF THE REQUIRED SOLUTION INVOLVES TWO EQUATIONS,SET ACC=1.,
*THE SOLUTION WILL BE IN TERMS OF A AND B.
*UNDEFINED TERMS ARE SET AUTOMATICALLY TO ZERO,UP
*TO A MAXIMUM OF 85.
*SECTION 4 IS THE PROGRAMMED SIMULATION.
*THE HYDRODYNAMIC COEFFICIENTS ARE HERE COMMON SINCE THE TWO
*SHIPS ARE ASSUMED IDENTICAL.
*SUBSCRIPT 1 REFERS TO THE REPLENISHING SHIP AND SUBSCRIPT 2
*REFERS TO THE RECEIVING SHIP.
*DDC IS THE ORDERED RUDDER ANGLE FOR COURSE CONTROL AND DDD
*IS THE ORDERED(HELM)RUDDER ANGLE FOR DISTANCE CONTROL.
*YI AND NI ARE THE INTERACTIVE FORCE AND MOMENT.
*SECTION 1 -COEFFICIENTS FOR A PARTICULAR SHIP
PARAM YI2=-0.000216
PARAM NI2=0.0000152
PARAM M=0.0045
PARAM IZ=0.0003
PARAM NR=-0.0012
PARAM NRD=-0.0002
PARAM NV=-0.0012
PARAM NVD=-0.0001
PARAM KB1=-0.00084
PARAM YR=0.004
PARAM YRD=-0.0002
PARAM YV=-0.0063
PARAM YVD=-0.0025
PARAM KA1=0.0019
PARAM XU=-0.0012
PARAM XUD=-0.00036
PARAM KC1=-0.0011
*SECTION 2 -PARAMETERS CALCULATIONS
AAA=M-YVD
BAA=-YV+10.0*(M-YVD)
ABA=-YRD
BBA=M-YR-10.0*YRD
AAB=-NVD
BAB=-NV-10.0*NVD
ABB=IZ-NRD
BBB=-NR+10.0*(IZ-NRD)
BCC=-XU+10.0*(M-XUD)
ACC=M-XUD
NC=-10.0*XU
GAA=-10.0*YV
GBA=10.0*(M-YR)
GAB=-10.0*NV

```



```

G8B=-10.0*NR
GCC=-10.0*XU
COFAA=ABB*ACC-ACB*ABC
COFAB=-(ABA*ACC-ACA*ABC)
COFAC=ABA*ACB-ABB*ACA
COFBA=-(AAB*ACC-ACB*AAC)
COFBB=AAA*ACC-ACA*AAC
COFBC=-(AAA*ACB-ACA*AAB)
COFCA=AAB*ABC-ABB*AAC
COFCB=-(AAA*ABC-ABA*AAC)
COFCC=AAA*ABB-ABA*AAB
S1=AAA*ABB*ACC+AAB*ABC*ACA+AAC*ACB*ABA
S2=ACA*ABB*AAC+ACB*ABC*AAA+ACC*AAB*ABA
DEL=S1-S2
*SECTION 3 -PHYSICAL DEFINITIONS
YAW2=B2
SWAY2=Y2
SURGE2=X2
*SECTION 4 -PROGRAMMED SIMULATION
ADDD2=(COFAA*I12+COFAB*I22+COFAC*I32)/DEL
ADD2=INTGRL(0.,ADDD2)
AD2=INTGRL(0.,ADD2)
A2=INTGRL(0.,AD2)
BDDD2=(COFBA*I12+COFBB*I22+COFBC*I32)/DEL
BDD2=INTGRL(0.,BDDD2)
BD2=INTGRL(0.,BDD2)
B2=INTGRL(0.,BD2)
CDDD2=(COFCA*I12+COFCB*I22+COFCC*I32)/DEL
CDD2=INTGRL(0.,CDDD2)
CD2=INTGRL(0.,CDD2)
C2=INTGRL(0.,CD2)
YDOT2=CD2*SIN(YAW2)+AD2*COS(YAW2)
Y2=INTGRL(0.,YDOT2)
XDOT2=CD2*COS(YAW2)-AD2*SIN(YAW2)
X2=INTGRL(0.,XDOT2)
I12=-BAA*ADD2-GAA*AD2-BBA*BDD2-GBA*BD2+IF12
I22=-BAB*ADD2-GAB*AD2-BBB*BDD2-GBB*BD2+IF22
I32=-BCC*CDD2-GCC*CD2+IF32
IF12=10.0*KA1*DD2+YI2
IF22=10.0*KB1*DD2+NI2
IF32=10.0*KC1*DD2+NC
*SECTION 5 -OUTPUT CHARACTERISTICS
PRTPLOT YAW2
PRTPLOT SWAY2
LABEL S3-LAT. DYNAMICS
TIMER FINTIM=40.0,DELT=0.02,PRDEL=0.2
END
STOP
ENDJOB

```


COMPUTER PROGRAM VI

```

*THIS PROGRAM SIMULATES THE DYNAMICS OF TWO SURFACE SHIPS.
*THE EQUATIONS OF MOTION FOR EACH ARE TO BE IN THE FORM:
*(AAAS3+BAAS2+GAAS+DAA)A+(ABAS3+BBAS2+GBAS+DBA)B+
*(ACAS3+BCAS2+GCAS+DCA)C=KA1*D1+KA2*D2+NA
*(AABS3+BABS2+GABS+DAB)A+(ABBS3+BBBS2+GBBS+DBB)B+
*(ACBS3+BCBS2+GCBS+DCB)C=KB1*D1+KB2*D2+NB
*(AACS3+BACS2+GACS+DAC)A+(ABCS3+BBCS2+GBCS+DBC)B+
*(ACCS3+BCCS2+GCCS+DCC)C=KC1*D1+KC2*D2+NC
*A,B AND C ARE THE VARIABLES.
*D1,D2 ARE THE DEFLECTIONS OF THE RUDDER(S).
*NA,NB,NC ARE TERMS IN WHICH ANY EXTERNAL FORCES OR MOMENTS,
*OR NONLINEAR TERMS CAN BE INCLUDED.
*AAB,BAB....,KA1...MUST BE DEFINED IN SECTION 2 AS FUNCTIONS
*OF THE HYDRODYNAMIC COEFFICIENTS,THE VALUES OF WHICH ARE
*INTRODUCED IN SECTION 1.
*NO ORDER IS REQUIRED.
*IN SECTION 3 THE VARIABLES ARE DEFINED,E.G. YAW=B.
*THE FIRST AND SECOND DERIVATIVES OF THE VARIABLES ARE:
*ADOT,BDOT,CBOT,ADDOT,BDDOT,CDDOT.
*IF THE REQUIRED SOLUTION INVOLVES TWO EQUATIONS,SET ACC=1.,
*THE SOLUTION WILL BE IN TERMS OF A AND B.
*UNDEFINED TERMS ARE SET AUTOMATICALLY TO ZERO,UP
*TO A MAXIMUM OF 85.
*SECTION 4 IS THE PROGRAMMED SIMULATION.
*THE HYDRODYNAMIC COEFFICIENTS ARE HERE COMMON SINCE THE TWO
*SHIPS ARE ASSUMED IDENTICAL.
*SUBSCRIPT 1 REFERS TO THE REPLENISHING SHIP AND SUBSCRIPT 2
*REFERS TO THE RECEIVING SHIP.
*DDC IS THE ORDERED RUDDER ANGLE FOR COURSE CONTROL AND DDD
*IS THE ORDERED(HELM)RUDDER ANGLE FOR DISTANCE CONTROL.
*YI AND NI ARE THE INTERACTIVE FORCE AND MOMENT.
*SECTION 1 -COEFFICIENTS FOR A PARTICULAR SHIP
PARAM K1=2.34
PARAM KT1=2.73
PARAM K2=2.35
PARAM KT2=2.75
PARAM KP2=1.1
PARAM KTP2=1.3
PARAM NI1=-0.0000152
PARAM YI1=0.000216
PARAM NI2=0.0000152
PARAM YI2=-0.000216
PARAM M=0.0045
PARAM IZ=0.0003
PARAM NR=-0.0012
PARAM NRD=-0.0002
PARAM NV=-0.0012
PARAM NVD=-0.0001
PARAM KB1=-0.00084
PARAM YR=0.004
PARAM YRD=-0.0002
PARAM YV=-0.0063
PARAM YVD=-0.0025
PARAM KA1=0.0019
PARAM XU=-0.0012
PARAM XUD=-0.00036
PARAM KC1=-0.0011
*SECTION 2 -PARAMETERS CALCULATIONS
AAA=M-YVD
BAA=-YV+10.0*(M-YVD)
ABA=-YRD
BBA=M-YR-10.0*YRD
AAB=-NVD
BAB=-NV-10.0*NVD

```



```

ABB=IZ-NRD
BBB=-NR+10.0*(IZ-NRD)
BCC=-XU+10.0*(M-XUD)
ACC=M-XUD
NC=-10.0*XU
GAA=-10.0*YV
GBA=10.0*(M-YR)
GAB=-10.0*NV
GBB=-10.0*NR
GCC=-10.0*XU
COFAA=ABB*ACC-ACB*ABC
COFAB=-(ABA*ACC-ACA*ABC)
COFAC=ABA*ACB-ABB*ACA
COFEA=-(AAB*ACC-ACB*AAAC)
COFBB=AAA*ACC-ACA*AAAC
COFBC=-(AAA*ACB-ACA*AAAB)
COFCA=AAB*ABC-ABB*AAAC
COFCB=-(AAA*ABC-ABA*AAAC)
COFCC=AAA*ABB-ABA*AAAB
S1=AAA*ABB*ACC+AAB*ABC*ACA+AAC*ACB*ABA
S2=ACA*ABB*AAAC+ACB*ABC*AAA+ACC*AAAB*ABA
DEL=S1-S2

```

*SECTION 3 -PHYSICAL DEFINITIONS

```

YAW1=R1
SWAY1=Y1
SURGE1=X1
YAW2=B2
SWAY2=Y2
SURGE2=X2

```

*SECTION 4 -PROGRAMMED SIMULATION

```

ADDD1=(COFAA*I11+COFAB*I21+COFAC*I31)/DEL
ADD1=INTGRL(0.,ADDD1)
AD1=INTGRL(0.,ADD1)
A1=INTGRL(0.,AD1)
BDDD1=(COFBA*I11+COFBB*I21+COFBC*I31)/DEL
BDD1=INTGRL(0.,BDDD1)
BD1=INTGRL(0.,BDD1)
B1=INTGRL(0.,BD1)
CDDD1=(COFCA*I11+COFCB*I21+COFCC*I31)/DEL
CDD1=INTGRL(0.,CDDD1)
CD1=INTGRL(0.,CDD1)
C1=INTGRL(0.,CD1)
YDOT1=CD1*SIN(YAW1)+AD1*COS(YAW1)
Y1=INTGRL(0.,YDOT1)
XDOT1=CD1*COS(YAW1)-AD1*SIN(YAW1)
X1=INTGRL(0.,XDOT1)
I11=-BAA*ADD1-GAA*AD1-BBA*BDD1-GBA*BD1+IF11
I21=-BAB*ADD1-GAB*AD1-BBB*BDD1-GBB*BD1+IF21
I31=-BCC*CDD1-GCC*CD1+IF31
IF11=10.0*KAI*DD1+YI1
IF21=10.0*KBI*DD1+NI1
IF31=10.0*KCI*DD1+NC
DDC1=K1*B1+KT1*BD1
DD1=DDC1+DDD1
ADDD2=(COFAA*I12+COFAB*I22+COFAC*I32)/DEL
ADD2=INTGRL(0.,ADDD2)
AD2=INTGRL(0.,ADD2)
A2=INTGRL(0.,AD2)
BDDD2=(COFBA*I12+COFBB*I22+COFBC*I32)/DEL
BDD2=INTGRL(0.,BDDD2)
BD2=INTGRL(0.,BDD2)
B2=INTGRL(0.,BD2)
CDDD2=(COFCA*I12+COFCB*I22+COFCC*I32)/DEL
CDD2=INTGRL(0.,CDDD2)
CD2=INTGRL(0.,CDD2)
C2=INTGRL(0.,CD2)
YDOT2=CD2*SIN(YAW2)+AD2*COS(YAW2)
Y2=INTGRL(0.,YDOT2)
XDOT2=CD2*COS(YAW2)-AD2*SIN(YAW2)
X2=INTGRL(0.,XDOT2)
I12=-3AA*ADD2-GAA*AD2-BBA*BDD2-GBA*BD2+IF12
I22=-BAB*ADD2-GAB*AD2-BBB*BDD2-GBB*BD2+IF22

```



```

I32=-BCC*CDD2-GCC*CD2+IF32
IF12=10.0*KA1*CD2+YI2
IF22=10.0*KB1*DD2+NI2
IF32=10.0*KC1*DD2+NC
D=Y2-Y1
DDOT=YDOT2-YDOT1
DDC2=K2*B2+KT2*BD2
DDD2=KP2*D+KTP2*DDOT
DD2=DDD2+DDC2
*SECTION 5 -OUTPUT CHARACTERISTICS
PRTPLOT SWAY1
PRTPLOT YAW1
PRTPLOT SURGE1
PRTPLOT SWAY2
PRTPLOT YAW2
PRTPLOT SURGE2
LABEL S3-LAT. DYNAMICS
TIMER FINTIM=40.0,DELT=0.02,PRDEL=0.2
END
STOP
ENDJOB

```


COMPUTER PROGRAM VII

```

*THIS PROGRAM SIMULATES THE DYNAMICS OF TWO SURFACE SHIPS.
*THE EQUATIONS OF MOTION FOR EACH ARE TO BE IN THE FORM:
*(AAAS3+BAAS2+GAAS+DAA)A+(ABAS3+BBAS2+GBAS+DBA)B+
*(ACAS3+BCAS2+GCAS+DCA)C=KA1*D1+KA2*D2+NA
*(AABS3+BABS2+GABS+DAB)A+(ABBS3+BBBS2+GBBS+DBB)B+
*(ACBS3+BCBS2+GCBs+DCB)C=KB1*D1+KB2*D2+NB
*(AACS3+BACS2+GACS+DAC)A+(ABCS3+BBCS2+GBCS+DBC)B+
*(ACCS3+BCCS2+GCCS+DCC)C=KC1*D1+KC2*D2+NC
*A, B AND C ARE THE VARIABLES.
*D1,D2 ARE THE DEFLECTIONS OF THE RUDDER(S).
*NA,NB,NC ARE TERMS IN WHICH ANY EXTERNAL FORCES OR MOMENTS,
*OR NONLINEAR TERMS CAN BE INCLUDED.
*AAB,BAB,...,KA1...MUST BE DEFINED IN SECTION 2 AS FUNCTIONS
*OF THE HYDRODYNAMIC COEFFICIENTS, THE VALUES OF WHICH ARE
*INTRODUCED IN SECTION 1.
*NO ORDER IS REQUIRED.
*IN SECTION 3 THE VARIABLES ARE DEFINED, E.G. YAW=B.
*THE FIRST AND SECOND DERIVATIVES OF THE VARIABLES ARE:
*ADOT,BDOT,CDOT,ADDDOT,BDDOT,CDDOT.
*IF THE REQUIRED SOLUTION INVOLVES TWO EQUATIONS, SET ACC=1.,
*THE SOLUTION WILL BE IN TERMS OF A AND B.
*UNDEFINED TERMS ARE SET AUTOMATICALLY TO ZERO, UP
*TO A MAXIMUM OF 85.
*SECTION 4 IS THE PROGRAMMED SIMULATION.
*THE HYDRODYNAMIC COEFFICIENTS ARE HERE COMMON SINCE THE TWO
*SHIPS ARE ASSUMED IDENTICAL.
*SUBSCRIPT 1 REFERS TO THE REPLENISHING SHIP AND SUBSCRIPT 2
*REFERS TO THE RECEIVING SHIP.
*DDC IS THE ORDERED RUDDER ANGLE FOR COURSE CONTROL AND DDD
*IS THE ORDERED(HELM)RUDDER ANGLE FOR DISTANCE CONTROL.
*YI AND NI ARE THE INTERACTIVE FORCE AND MOMENT.
*SECTION 1 -COEFFICIENTS FOR A PARTICULAR SHIP
PARAM K1=2.3
PARAM KT1=2.7
PARAM K2=2.35
PARAM KT2=2.75
PARAM KP2=1.1
PARAM KTP2=1.3
PARAM NI1=-0.0000152
PARAM YI1=0.000216
PARAM NI2=0.0000152
PARAM YI2=-0.000216
PARAM M=0.0045
PARAM IZ=0.0003
PARAM NR=-0.0012
PARAM NRD=-0.0002
PARAM NV=-0.0012
PARAM NVD=-0.0001
PARAM KB1=-0.00084
PARAM YR=0.004
PARAM YRD=-0.0002
PARAM YV=-0.0063
PARAM YVD=-0.0025
PARAM KA1=0.0019
PARAM XU=-0.0012
PARAM XUD=-0.00036
PARAM KC1=-0.0011
*SECTION 2 -PARAMETERS CALCULATIONS
AAA=M-YVD
BAA=-YV+10.0*(M-YVD)
ABA=-YRD
BBA=M-YR-10.0*YRD
AA3=-NVD
BAB=-NV-10.0*NVD

```



```

ABB=IZ-NRD
BBB=-NR+10.0*(IZ-NRD)
BCC=-XU+10.0*(M-XUD)
ACC=M-XUD
NC=-10.0*XU
GAA=-10.0*YV
GBA=10.0*(M-YR)
GAB=-10.0*NV
GBB=-10.0*NR
GCC=-10.0*XU
COFAA=ABB*ACC-ACB*ABC
COFAB=-(ABA*ACC-ACA*ABC)
COFAC=ABA*ACB-ABB*ACA
COFBA=-(AAB*ACC-ACB*AAC)
COFBB=AAA*ACC-ACA*AAC
COFBC=-(AAA*ACB-ACA*AAB)
COFCA=AAB*ABC-ABB*AAC
COFCB=-(AAA*ABC-ABA*AAC)
COFCC=AAA*ABB-ABA*AAB
S1=AAA*ABB*ACC+AAB*ABC*ACA+AAC*ACB*ABA
S2=ACA*ABB*AAC+ACB*ABC*AAA+ACC*AAB*ABA
DEL=S1-S2

```

*SECTION 3 -PHYSICAL DEFINITIONS

```

YAW1=B1
SWAY1=Y1
SURGE1=X1
YAW2=B2
SWAY2=Y2
SURGE2=X2

```

*SECTION 4 -PROGRAMMED SIMULATION

```

ADDD1=(COFAA*I11+COFAB*I21+COFAC*I31)/DEL
ADD1=INTGRL(0.,ADDD1)
AD1=INTGRL(0.,ADD1)
A1=INTGRL(0.,AD1)
BDD1=(COFBA*I11+COFBB*I21+COFBC*I31)/DEL
BDD1=INTGRL(0.,BDD1)
BD1=INTGRL(0.,BDD1)
B1=INTGRL(0.,BD1)
CDD1=(COFCA*I11+COFCB*I21+COFCC*I31)/DEL
CDD1=INTGRL(0.,CDD1)
CD1=INTGRL(0.,CDD1)
C1=INTGRL(0.,CD1)
YDOT1=CD1*SIN(YAW1)+AD1*COS(YAW1)
Y1=INTGRL(0.,YDOT1)
XDOT1=CD1*COS(YAW1)-AD1*SIN(YAW1)
X1=INTGRL(0.,XDOT1)
I11=-BAA*ADD1-GAA*AD1-BBA*BDD1-GBA*BD1+IF11
I21=-BAB*ADD1-GAB*AD1-BBB*BDD1-GBB*BD1+IF21
I31=-BCC*CDD1-GCC*CD1+IF31
IF11=10.0*KA1*DD1+Y11
IF21=10.0*KB1*DD1+NI1
IF31=10.0*KC1*DD1+NC
DDC1=K1*B1+KT1*BD1
DD1=DDC1+DDD1
ADDD2=(COFAA*I12+COFAB*I22+COFAC*I32)/DEL
ADD2=INTGRL(0.,ADDD2)
AD2=INTGRL(0.,ADD2)
A2=INTGRL(0.,AD2)
BDD2=(COFBA*I12+COFBB*I22+COFBC*I32)/DEL
BDD2=INTGRL(0.,BDD2)
BD2=INTGRL(0.,BDD2)
B2=INTGRL(0.,BD2)
CDD2=(COFCA*I12+COFCB*I22+COFCC*I32)/DEL
CDD2=INTGRL(0.,CDD2)
CD2=INTGRL(0.,CDD2)
C2=INTGRL(0.,CD2)
YDOT2=CD2*SIN(YAW2)+AD2*COS(YAW2)
Y2=INTGRL(0.,YDOT2)
XDOT2=CD2*COS(YAW2)-AD2*SIN(YAW2)
X2=INTGRL(0.,XDOT2)
I12=-BAA*ADD2-GAA*AD2-BBA*BDD2-GBA*BD2+IF12
I22=-BAB*ADD2-GAB*AD2-BBB*BDD2-GBB*BD2+IF22

```



```

I32=-BCC*CDD2-GCC*CD2+IF32
IF12=10.0*KAI*DD2+YI2
IF22=10.0*KB1*DD2+NI2
IF32=10.0*KC1*DD2+NC
D=Y2-Y1
DDOT=YDOT2-YDOT1
DDC2=K2*B2+KT2*BD2
DDD2=KP2*D+KTP2*DDGT
DD2=DDD2+DDC2
*SECTION 5 -OUTPUT CHARACTERISTICS
PRTPLCT SWAY1
PRTPLCT YAW1
PRTPLCT SURGE1
PRTPLCT SWAY2
PRTPLCT YAW2
PRTPLCT SURGE2
LABEL S3-LAT. DYNAMICS
TIMER FINTIM=40.0,DELT=0.02,PRDEL=0.2
END
STOP
ENDJOB

```


COMPUTER PROGRAM VIII

```

*THIS PROGRAM SIMULATES THE DYNAMICS OF TWO SURFACE SHIPS.
*THE EQUATIONS OF MOTION FOR EACH ARE TO BE IN THE FORM:
*(AAAS3+BAAS2+GAAS+DAA)A+(ABAS3+BBAS2+GBAS+DBA)B+
*(ACAS3+BCAS2+GCAS+DCA)C=KA1*D1+KA2*D2+NA
*(AABS3+BABS2+GABS+DAB)A+(ABBS3+BBBS2+GBBS+DBB)B+
*(ACBS3+BCBS2+GCBs+DCB)C=KB1*D1+KB2*D2+NB
*(AACS3+BACS2+GACS+DAC)A+(ABCS3+BBCS2+GBCS+DBC)B+
*(ACCS3+BCCS2+GCCS+DCC)C=KC1*D1+KC2*D2+NC
*A,B AND C ARE THE VARIABLES.
*D1,D2 ARE THE DEFLECTIONS OF THE RUDDER(S).
*NA,NB,NC ARE TERMS IN WHICH ANY EXTERNAL FORCES OR MOMENTS,
*OR NONLINEAR TERMS CAN BE INCLUDED.
*AAB,BAB,...,KA1...MUST BE DEFINED IN SECTION 2 AS FUNCTIONS
*OF THE HYDRODYNAMIC COEFFICIENTS,THE VALUES OF WHICH ARE
*INTRODUCED IN SECTION 1.
*NO ORDER IS REQUIRED.
*IN SECTION 3 THE VARIABLES ARE DEFINED,E.G. YAW=B.
*THE FIRST AND SECOND DERIVATIVES OF THE VARIABLES ARE:
*ADOT,BDOT,CBOT,ADDOT,BDDOT,CDDOT.
*IF THE REQUIRED SOLUTION INVOLVES TWO EQUATIONS,SET ACC=1.,
*THE SOLUTION WILL BE IN TERMS OF A AND B.
*UNDEFINED TERMS ARE SET AUTOMATICALLY TO ZERO,UP
*TO A MAXIMUM OF 85.
*SECTION 4 IS THE PROGRAMMED SIMULATION.
*THE HYDRODYNAMIC COEFFICIENTS ARE HERE COMMON SINCE THE TWO
*SHIPS ARE ASSUMED IDENTICAL.
*SUBSCRIPT 1 REFERS TO THE REPLENISHING SHIP AND SUBSCRIPT 2
*REFERS TO THE RECEIVING SHIP.
*DDC IS THE ORDERED RUDDER ANGLE FOR COURSE CONTROL AND DDD
*IS THE ORDERED(HELM)RUDDER ANGLE FOR DISTANCE CONTROL.
*YI AND NI ARE THE INTERACTIVE FORCE AND MOMENT.
*SECTION 1 -COEFFICIENTS FOR A PARTICULAR SHIP
PARAM K1=2.35
PARAM KT1=2.75
PARAM KP1=1.1
PARAM KTP1=1.3
PARAM K2=2.35
PARAM KT2=2.75
PARAM KP2=1.1
PARAM KTP2=1.3
PARAM NI1=-0.0000152
PARAM YI1=0.000016
PARAM NI2=0.0000152
PARAM YI2=-0.0000216
PARAM M=0.0045
PARAM IZ=0.0003
PARAM NR=-0.0012
PARAM NRD=-0.0002
PARAM NV=-0.0012
PARAM NVD=-0.0001
PARAM KB1=-0.00084
PARAM YR=0.004
PARAM YRD=-0.0002
PARAM YV=-0.0063
PARAM YVD=-0.0025
PARAM KA1=0.0019
PARAM XU=-0.0012
PARAM XUD=-0.00036
PARAM KC1=-0.0011
*SECTION 2 -PARAMETERS CALCULATIONS
AAA=M-YVD
BAA=-YV+10.0*(M-YVD)
ABA=-YRD
BBA=M-YR-10.0*YRD

```



```

AAB=-NVD
BAB=-NV-10.0*NVD
ABB=IZ-NRD
BBB=-NR+10.0*(IZ-NRD)
BCC=-XU+10.0*(M-XUD)
ACC=M-XUD
NC=-10.0*XU
GAA=-10.0*YV
GBA=10.0*(M-YR)
GAB=-10.0*NV
GBB=-10.0*NR
GCC=-10.0*XU
COFAA=ABB*ACC-ACB*ABC
COFAB=-(ABA*ACC-ACA*ABC)
COFAC=ABA*ACB-ABB*ACA
COFBA=-(AAB*ACC-ACB*AAC)
COFBB=AAA*ACC-ACA*AAC
COFBC=-(AAA*ACB-ACA*AAB)
COFCA=AAB*ABC-ABB*AAC
COFCB=-(AAA*ABC-ABA*AAC)
COFCC=ABA*ABB-ABA*AAB
S1=AAA*ABB*ACC+AAB*ABC*ACA+AAC*ACB*ABA
S2=ACA*ABB*AAC+ACB*ABC*AAA+ACC*AAB*ABA
DEL=S1-S2

```

*SECTION 3 -PHYSICAL DEFINITIONS

```

YAW1=B1
SWAY1=Y1
SURGE1=X1
YAW2=B2
SWAY2=Y2
SURGE2=X2

```

*SECTION 4 -PROGRAMMED SIMULATION

```

ADDD1=(COFAA*I11+COFAB*I21+COFAC*I31)/DEL
ADD1=INTGRL(0.,ADDD1)
AD1=INTGRL(0.,ADD1)
A1=INTGRL(0.,AD1)
BDD1=(COFBA*I11+COFBB*I21+COFBC*I31)/DEL
BDD1=INTGRL(0.,BDD1)
BD1=INTGRL(0.,BDD1)
B1=INTGRL(0.,BD1)
CDD1=(COFCA*I11+COFCB*I21+COFCC*I31)/DEL
CDD1=INTGRL(0.,CDD1)
CD1=INTGRL(0.,CDD1)
C1=INTGRL(0.,CD1)
YDOT1=CD1*SIN(YAW1)+AD1*COS(YAW1)
Y1=INTGRL(0.,YDOT1)
XDOT1=CD1*COS(YAW1)-AD1*SIN(YAW1)
X1=INTGRL(0.,XDOT1)
I11=-BAA*ADD1-GAA*AD1-BBA*BDD1-GBA*BD1+IF11
I21=-BAB*ADD1-GAB*AD1-BBB*BDD1-GBB*BD1+IF21
I31=-BCC*CDD1-GCC*CD1+IF31
IF11=10.0*KAI*DD1+YI1
IF21=10.0*KBI*DD1+NI1
IF31=10.0*KCI*DD1+NC
DDC1=K1*B1+KT1*BD1
DD1=-KP1*D-KT1*DDOT
DD1=DDC1+DD1
ADDD2=(COFAA*I12+COFAB*I22+COFAC*I32)/DEL
ADD2=INTGRL(0.,ADDD2)
AD2=INTGRL(0.,ADD2)
A2=INTGRL(0.,AD2)
BDD2=(COFBA*I12+COFBB*I22+COFBC*I32)/DEL
BDD2=INTGRL(0.,BDD2)
BD2=INTGRL(0.,BDD2)
B2=INTGRL(0.,BD2)
CDD2=(COFCA*I12+COFCB*I22+COFCC*I32)/DEL
CDD2=INTGRL(0.,CDD2)
CD2=INTGRL(0.,CDD2)
C2=INTGRL(0.,CD2)
YDOT2=CD2*SIN(YAW2)+AD2*COS(YAW2)
Y2=INTGRL(0.,YDOT2)
XDOT2=CD2*COS(YAW2)-AD2*SIN(YAW2)

```



```

X2=INTGRL(0.,XDOT2)
I12=-BAA*ADD2-GAA*AD2-BBA*BDD2-GBA*BD2+IF12
I22=-BAB*ADD2-GAB*AD2-BBB*BDD2-GBB*BD2+IF22
I32=-BCC*CDD2-GCC*CD2+IF32
IF12=10.0*KA1*DD2+YI2
IF22=10.0*KB1*DD2+NI2
IF32=10.0*KC1*DD2+NC
D=Y2-Y1
DDOT=YDOT2-YDOT1
DDC2=K2*B2+KT2*3D2
DDD2=KP2*D+KTP2*DDOT
DD2=DDD2+DDC2
*SECTION 5 -OUTPUT CHARACTERISTICS
PRTPLOT SWAY1
PRTPLOT YAW1
PRTPLOT SURGE1
PRTPLOT SWAY2
PRTPLOT YAW2
PRTPLOT SURGE2
LABEL S3-LAT. DYNAMICS
TIMER FINTIM=40.0,DELT=0.02,PRDEL=0.2
END
STOP
ENDJOB

```


LIST OF REFERENCES

1. Taylor, D. W., Some Model Experiments on Suction of Vessels, Transactions, Society of Naval Architects and Mechanical Engineers, 1909.
2. Silverstein, B. L., Linearized Theory of the Interaction of Ships, University of California, Institute of Engineering Research, May 1957.
3. DTMB Report 1461, Interaction Effects Between Ships Close Aboard in Deep Water, by R. N. Newton, p. 2-19, October 1960.
4. Abcowitz, A. M., Stability and Motion Control of Ocean Vehicles, p. 35, p. I-92, The MIT Press, 1969.
5. Mandel, P., and others, Principles of Naval Architecture, p. 468, Society of Naval Architects and Marine Engineers, 1967.

INITIAL DISTRIBUTION LIST

	No. Copies
1. Defense Documentation Center Cameron Station Alexandria, Virginia 22314	2
2. Library, Code 0212 Naval Postgraduate School Monterey, California 93940	2
3. Professor George J. Thaler, 52Tr Department of Electrical Engineering Naval Postgraduate School Monterey, California 93940	6
4. Hellenic Navy Command Paparigopoulou 2 Athens, Greece	1
5. LT Theodore Sarzetakis, H.N. H. S. Sfendoni Hellenic Navy Command Paparigopoulou 2 Athens, Greece	1
6. Samuel H. Brown NSRDL Annapolis, Maryland	1
7. Larry Puckett NSRDL Annapolis, Maryland	1
8. Walt Blumberg NSRDL Annapolis, Maryland	1
9. Professor M. L. Wilcox, Code 52Tx Department of Electrical Engineering Naval Postgraduate School Monterey, California 93940	1
10. Professor A. Gerba, Code 52Gz Department of Electrical Engineering Naval Postgraduate School Monterey, California 93940	1

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)

Naval Postgraduate School
Monterey, California 93940

2a. REPORT SECURITY CLASSIFICATION

Unclassified

2b. GROUP

3. REPORT TITLE

MANEUVERING CONTROL OF REPLENISHMENT AT SEA

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Master's Thesis; September 1972

5. AUTHOR(S) (First name, middle initial, last name)

Theodoros Sarzetakis
Lieutenant, Hellenic Navy

6. REPORT DATE

September 1972

7a. TOTAL NO. OF PAGES

111

7b. NO. OF REFS

5

8a. CONTRACT OR GRANT NO

b. PROJECT NO.

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY

Naval Postgraduate School
Monterey, California 93940

13. ABSTRACT

An investigation of the maneuvering control of ships involved in the replenishment at sea operation under calm water conditions is carried out.

The linearized differential equations of motion of a vessel in the horizontal plane are established and implemented for the formation of computer programs, useful for the study of the behavior and stability of the ship with and without the influence of control surfaces (rudders).

Three methods of controlling automatically the maneuvering of two ships, in replenishment at sea, under the influence of interactive forces and moments, based on the classical feedback control theory are presented, compared and conclusions are finally drawn about the efficiency of these methods.

14

KEY WORDS

LINK A

LINK B

LINK C

ROLE

WT

ROLE

WT

ROLE

WT

Replenishment at sea
 Maneuvering control
 Path keeping control
 Distance keeping control
 Interaction Effects

Thesis
S168
c.1

Sarzetakis
Maneuvering control
of replenishment at sea

138058

20 APR 73
VZ BYN S

15 DEC 79

21 JAN 81

19 MAY 87

21031
21491

25210

26819

33307

Thesis
S168
c.1

Sarzetakis
Maneuvering control
of replenishment at sea.

138058

thesS168

Maneuvering control of replenishment at



3 2768 001 00167 0

DUDLEY KNOX LIBRARY